**HYDRAULIC MACHINES**

**MODULE – 1**

**IMPACT OF JET**

**IMPACT OF JET**

When the jet impinges on plates or vanes, its momentum is changed and a hydrodynamic force is exerted, this exerted force is known as impact of jet.

**JET**

The jet is a stream of liquid comes out from nozzle with a high velocity under constant pressure.

**INTRODUCTION**

The impulse momentum principle is used to calculate the hydrodynamic force of jet on the vane. This principle is derived from the Newton’s IInd Law of motion. The hydrodynamic force is due to the change in the momentum of the jet as a consequence of the impact. This force is equal to the rate of change of momentum. i.e., the force is equal to (mass striking the plate per second) x (Change in velocity).

From Newton’s 2nd law, F = ma = m (V1 - V2)

t

**Impulse** of a force is given by the change in momentum caused by the force on the body.

Ft = mV1 - mV2 = Initial Momentum - Final Momentum

Force exerted by jet on the plate in the direction of jet, F = m (V1 - V2)/t

= (Mass/Time) (Initial Velocity - Final Velocity)

= ꝭQ (V1 - V2)

 F = ꝭa V (V1 - V2)

1. **Force exerted by the jet on a stationary plate.**
2. Plate is vertical to the jet.
3. Plate is inclined to the jet.
4. Plate is curved.
5. **Force exerted by the jet on a moving plate.**

a) Plate is vertical to the jet.

b) Plate is inclined to the jet.

c) Plate is curved.

**Assumptions were made while analyzing impact of jet**

* Plate is smooth.
* No loss of energy due to impact of jet.
* Jet will move over the plate after striking with a velocity equal to initial velocity.
* Uniform distribution of velocity.
* Pressure everywhere is atmosphere.
1. **Force exerted by the jet on a stationary vertical plate**

Consider jet of water coming out from the nozzle strikes the vertical plate,



Let,

V = velocity of jet, d = diameter of the jet in meters,

a = area of cross-section of the jet, in m2

ρ - is the density of the fluid, kg/m3

The force exerted by the jet on the plate in the direction of jet.

Fx = Rate of change of momentum in the direction of force.

Rate of change of momentum in the direction of force,

 = (initial momentum – final momentum)/time

= (mass x initial velocity – mass x final velocity) / time

= mass/time (initial velocity – final velocity)

= mass/ sec x (velocity of jet before striking - velocity of jet after striking)

= m (V-0)

= ρaV (V-0)

 **Fx = ρaV2**

1. **Force exerted by the jet on a stationary inclined plate**

If the surface is inclined at an angle to the jet, as shown in Fig, the jet velocity can be resolved into two components, one normal to the surface and other parallel to it. Since, water leaves the surface tangentially; there is no component of force in that direction after impinging.

Applying impulse momentum equation in the direction normal to the plate, then normal force on the plate,

**Fn = Mass of jet striking per second (initial velocity of the jet before striking - final velocity of the jet after striking)**

 **= m (V1n - V2n)**

 **= ρaV (V sin θ - 0)**

**Fn = ρaV2 sin θ**

This force can be resolved into two components, one in the direction of the jet and other normal to the direction of flow.



Then Force exerted by the jet in the direction of flow,

**Fx = Fn Cos (90-θ)**

 **= Fn Sin θ**

 **= ρaV2 sin2θ**

Force exerted by the jet in the direction normal to the flow,

**Fy = Fn Sin (90-θ)**

 **= Fn Cos θ**

 **= ρaV2 Sin θ Cos θ**

1. **Force exerted by the jet on a stationary cuved plate**
2. **Impact of jets on stationary curved plates at the center**

The impact of jet on the curved surface is of much practical significance. The jet of water can be introduced tangentially, whereas in other case it can strikes at some angle to the entrance portion of the surface. Here, both such cases deserve individual consideration.

Consider the case when the jet strikes horizontally at the centre of the vane on the concave side as shown in figure below.



 After it strikes, it gets divided, glides over the surface and leaves the vane tangentially with same velocity ‘v’. The velocity at outlet of the plate can be resolved into two components, one in the direction of the jet and the other perpendicular to the direction of the jet.

Component of velocity in the direction of the jet is given by, −(V cosθ) here -ve sign has been considered because; outlet velocity is in the opposite direction of the jet of water coming out of from nozzle. Component of velocity perpendicular to the jet is given by, V sinθ

V1X is the initial velocity of the jet before striking in the direction of the jet,

 ie in the X direction = V

V2X =is the final velocity of the jet after striking in the direction of jet, ie in the X - direction = -V cosθ

V1Y = is the initial velocity of the jet striking in the direction of Y = 0

V2Y = is the final velocity of the jet after striking in the direction of Y = V Sinθ

**Applying impulse-momentum equation in the direction normal to the plate, then normal force on the plate,**

Force exerted by the jet in the direction of the jet,

FX = Mass of jet striking per second (Initial velocity of the jet before striking - Final velocity of the jet after striking)

 = m (V1x - V2x)

 = ρaV [V- (-V Cosθ)]

 = ρaV [V + V Cosθ]

**Fx = ρaV2 [1+ Cosθ ]**

Similarly, Force exerted by the jet normal to the direction of the jet,

Fy = Mass of jet striking per second (initial velocity of the jet before striking - Final velocity of the jet after striking)

 = m (V1y - V2y )

 = ρaV [ 0 -V Sinθ ]

**Fy = - ρaV2 Sinθ**

Negative sign means the force is acting in the downward direction.

In this case angle of deflection of the jet = 180-θ

**Problem:** A jet of water of diameter 0.10m moving with a velocity of 20m/s, strikes acurved fixed symmetrical plate at the centre. Find the force exerted by the jet of water inthe direction of the jet, if the jet is deflected through an angle of 1200at the outlet of thecurved plate.

**Solution:** Given data,

Diameter of the jet, d= 0.10m;

Area of the jet, a= 0.00785m2;

Velocity of the jet, v=20m/s;

Angle of deflection = 1200; θ=600(because, angle of deflection=(1800 - θ)

Using the relation, for force exerted by the jet on the curved plate in the direction of the

jet,

**Fx= ρaV2 [ 1+ Cosθ ]**

 = 1000 x 0.00785 x 20 {1 + cos 60} = **4710 N**

1. **FORCE EXERTED BY THE JET ON A STATIONARY CURVED PLATE AT ONE END TANGENTIALLY**

Consider a jet of fluid strikes a fixed curved vane at one end glides over the vane and then leaves it tangentially. The velocity of the jet at the outlet will be same as that of inlet end.

**Case I: When the plate is symmetrical.**

Let the jet strikes the curved fixed plate symmetrical about X-axis at one end tangentially as shown in Fig. The plate is symmetrical, and then the angle made by the tangents at the two ends of the plate will be same.



 **Fig. Jet strikes on a stationary symmetrical curved plate/vane at one end tangentially.**

Let,

V = velocity of jet, d = diameter of the jet in meters,

 a = area of cross-section of the jet, in m2

 ρ - is the density of the fluid, kg/m3

 θ - is the angle made by the jet with X-axis at inlet tip of the curved plate.

 m - is the mass flow rate in kg/sec

 V1X - is the initial velocity of the jet before striking in the direction of the jet,

 ie in the X direction = Vcosθ

 V2X - is the final velocity of the jet after striking in the direction of jet,

 ie in the X - direction =  **-**V cosθ

 V1Y - is the initial velocity of the jet striking in the direction of Y = V Sinθ

 V2Y - is the final velocity of the jet after striking in the direction of Y = V Sinθ

Applying impulse-momentum equation in the direction normal to the plate, then normal force on the plate,

Force exerted by the jet in the direction of X,

Fx = Mass of jet striking per second (Initial velocity of the jet before striking - Final velocity of

 the jet after striking)

 = m(V1x - V2x)

 = ρaV [ V Cos θ - (-V Cos θ) ]

 = ρaV [ V Cos θ + V Cos θ) ]

 Fx = 2ρaV2 Cos θ

Similarly, Force exerted by the jet in the direction of Y,

Fy = Mass of jet striking per second (initial velocity of the jet before striking - Final velocity of the jet after striking)

 = m (V1y - V2y )

 = ρaV [ V Sin θ -V Sin θ ]

 = 0

**Case II: When the plate is unsymmetrical**

Let the jet strikes the curved fixed plate unsymmetrical about X-axis at one end tangentially, then the angle made by the tangents drawn at the inlet and outlet tips of the plate with X-axis will be different as shown in Fig.

Let,

 V = velocity of jet, d = diameter of the jet in meters,

 a = area of cross-section of the jet, in m2

 ρ - is the density of the fluid, kg/m3

 θ - is the angle made by the jet with X-axis at inlet tip of the curved plate.

 Φ - is the angle made by the jet with X-axis at outlet tip of the curved plate.

 m - is the mass flow rate in kg/sec

 V1X is the initial velocity of the jet before striking in the direction of the jet,

 ie in the X direction, V1X = V Cosθ

 V2X - is the final velocity of the jet after striking in the direction of jet,

ie in the X - direction , V2X = -V cosΦ

 V1Y  - is the initial velocity of the jet striking in the direction of Y = V Sinθ

 V2Y  - is the final velocity of the jet after striking in the direction of Y = V SinΦ

**Applying impulse-momentum equation in the direction normal to the plate, then normal force on the plate,**

Force exerted by the jet in the direction of X,

Fx = Mass of jet striking per second (Initial velocity of the jet before striking - Final velocity of the jet after striking)

 = m (V1x - V2x)

 = ρaV [ V Cosθ - (-V CosΦ) ]

 = ρaV [ V Cosθ + V CosΦ) ]

 **Fx = ρaV2 [Cosθ + CosΦ]**

Similarly, Force exerted by the jet in the direction of Y,

Fy = Mass of jet striking per second (initial velocity of the jet before striking - Final velocity of the jet after striking)

= m (V1y - V2y )

= ρaV [ V Sinθ -V SinΦ ]

 **Fy = ρaV2 [ Sinθ - SinΦ ]**

[Total angle of deflection of the jet = 180-(θ+Φ)]

**FORCE EXERTED BY THE JET ON MOVING PLATES**

Consider a jet of fluid strikes on a smooth vertical plate at the centre which is moving in the same direction as of jet, say with a velocity of u.



**Fig. Fluid jet striking on a moving vertical plate**

Let,

V - is the velocity of the jet measured in m/s

a - is the cross sectional area of the jet

u - is the velocity of the flat plate as a result of impact of jet

In this case jet strikes with a relative velocity, Vr = (V-u)

m - Mass of fluid striking the plate per second is given by = ρa(V-u)

**Force exerted by the jet on the moving plate in the direction of the jet, X-direction,**

Fx  = Mass of fluid striking per second (Initial velocity with which fluid strikes - Final velocity)

 = m [(V-u) - 0]

 = ρa (V-u) [(V-u)-0]

 Fx = ρa (V-u)2

In case of moving plate, work is done on the plate by the jet,

W = Force x Distance moved by the plate in the direction of force per second

 = Fx x u

 = ρa(V-u)2 x u Nm/s

But Work done per second is the power developed.

P = ρa(V-u)2 x u watts

Energy supplied by jet = Kinetic energy of issuing jet

 = 1/2 mV2

 = 1/2 ρaV x V2

 = 1/2 ρaV3

Efficiency of the jet, η = Output of the jet per second

 Input of the jet per second

 = Work done per second

 Energy supplied per second

 = ρa(V-u)2 x u

 1/2 ρaV3

 **η = 2 (V-u)2 u**

 **V*3***

**Problem:** A jet of water of diameter 50mm strikes a flat plate normally with a velocity of 26 m/s. The plate is moving with a velocity of 10 m/s in the direction of the jet and away from the jet. Find i) the force exerted by the jet on the plate. ii) Work done by the jet on the plate per second. iii) Power of the jet. iv) Efficiency of the jet.

Solution:- Given data,

Diameter of the jet, d = 50 mm = 50 x 10-3 m

Velocity of the jet , v = 26 m/s

Velocity of the plate, u = 10 m/s

Force exerted by the jet on the flat moving plate,

 Fx = ρa(V-u)2

 = 1000 x Π/4 x (50 x 10-3) x (26 - 10)2

 **= 502.65 N**

Work done by the jet on the plate per second,

 W = Fx x u

 = 502.65 x 10

 = **5026.5 Nm/s**

Power of the jet, P = Work done by the jet on the plate per second

 P = 5026.5 W

Efficiency of the jet, η = 2 (V-u)2 u

V*3*

η*=*2 (26 - 10)2 x 10 = **29.13 %**

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**FORCE EXERTED BY THE JET ON A FLAT INCLINED PLATE MOVING IN THE DIRECTION OF JET**

Consider a jet of fluid strikes on a smooth inclined flat plate at the centre which is moving in the same direction as of jet, say with a velocity of u.



**Fig. Fluid jet striking on a moving inclined plate**

Let,

V - is the velocity of the jet measured in m/s

 a - is the cross sectional area of the jet

 u - is the velocity of the flat plate as a result of impact of jet

 In this case jet strikes with a relative velocity, Vr = (V-u)

 m - mass of fluid striking the plate per second is given by = ρa(V-u)

 Force exerted by the jet of fluid on the moving plate in the direction normal to the plate,

Fn = Mass of fluid striking per second **(** Initial velocity in the normal direction with which fluid strike - Final velocity in the direction normal to the plate**)**

 **=** m[ (V-u) sinθ - 0]

 = ρa(V-u) [ (V-u) Sinθ - 0]

 = ρa(V-u)2 sinθ

The normal force Fn can be resolved into two components Fx and Fy in the direction of jet and perpendicular to the direction of the jet respectively.

**Fx** = Component of the normal force Fn parallel to the jet

 = Fn sinθ = ρa(V-u)2 sinθ x sinθ

 **= ρa(V-u)2 sin2θ**

 **Fy**  = Component of the normal force Fn perpendicular to the jet

 = Fn cosθ = ρa(V-u)2 sinθ x cosθ

 **= ρa(V-u)2 sinθ cosθ**

Work done by the jet on the plate per unit time, W = Fx x u

 **= ρa(V-u)2 sin2θ x u Nm/s**

Power developed, P = **ρa(V-u)2 sin2θ x u watts**

Energy supplied per second = 1 m V2

 2

 = 1 ρaV x V2 = 1 ρaV3

 2 2

Efficiency of the jet is the ratio of output of the jet per second to the input of the jet per second.

Efficiency,η = Output of the jet per second

Input of the jet per second

= Work done per second

Energy supplied per second

 = ρa(V-u)2 sin2θ x u

 1 ρaV3

 2

 **η = 2(V-u)2 sin2θ u**

 **V3**

**Problem:**A 75 mm diameter water jet having a velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45o to the axis of the jet. Find the normal pressure on the plate (i) When the plate is stationary (ii) When the plate is moving with a velocity of 15 m/s and away from the jet. Also determine the power and efficiency of the jet when the plate is moving.

Given data,

Diameter of the jet, d = 75 mm = 0.075 m

Absolute velocity of the jet, V = 30 m/s

Angle between jet and plate, θ = 450

Velocity of the plate, u = 15 m/s

1. When the plate is stationary, normal force on the plate,

 Fn = ρaV2 sinθ

 = 1000 x Π/4 x (0.075)2 x 302 x sin45

 = 2811.51 N

1. When the plate is moving with a velocity 15 m/s and away from the jet, the normal force on the plate,

 Fn = ρa(V-u)2 sinθ

 = 1000 x Π/4 x (0.075)2 x (30 - 15)2 x sin45

 = **702.88 N**

Force in the direction of the jet, Fx = Fn sinθ

 = 702.88 x sin45

 = **497 N**

Workdone per second, W = Fx x u = 497 x 15 = **7455 Nm/s**

Power, P = 7455 W = **7.5 kW**

Efficiency, η = 2 (V-u)2 sin2θ u

 V3

 = 2 x (300-15)2 sin245 x 15

303

 = **12.5 %**

**FORCE EXERTED BY THE JET STRIKING AT CENTRE 0N A CUVED PLATE / VANE WHEN THE PLATE IS MOVING IN THE DIRECTION OF THE JET.**

Consider a jet of fluid strikes at the centre of a symmetrical smooth curved plate moving with a uniform velocity in the direction of jet as shown in diagram.



Let,

 V = velocity of jet, d = diameter of the jet in meters,

 a = area of cross-section of the jet, in m2

 ρ- is the density of the fluid, kg/m3

 θ- is the angle made by the jet with X-axis at inlet tip of the curved plate.

 m - is the mass flow rate in kg/sec

 V1X is the initial velocity of the jet before striking in the direction of the jet,

 ie in the X direction = (V- u) cosθ

 V2X =is the final velocity of the jet after striking in the direction of jet,

 ie in the X - direction = -(V- u) cosθ

 V1Y = is the initial velocity of the jet striking in the direction of Y = (V- u) Sinθ

 V2Y = is the final velocity of the jet after striking in the direction of Y = (V-u) Sinθ

Applying impulse-momentum equation in the direction normal to the plate, then normal force on the plate,

Force exerted by the jet in the direction of X,

FX= Mass of jet striking per second (Initial velocity of the jet before striking - Final velocity of the jet after striking)

= m(V1x - V2x)

= ρa(V-u) [ (V-u) - (-(V-u) Cosθ) ]

= ρa(V- u) [ (V- u) + (V-u) Cosθ) ]

= ρa(V- u)2 [1 + Cosθ]

Similarly, Force exerted by the jet in the direction of Y,

Fy = Mass of jet striking per second (initial velocity of the jet before striking - Final velocity of the jet after striking)

= m (V1y - V2y )

= ρa(V- u) [ (V- u) Sinθ - (V- u) Sinθ ]

= 0

Workdone by the jet on the curved plate per second, W = Fx x u

 **W = ρa(V- u)2 [1 + Cos θ] u**

Power developed, **P = ρa (V- u) 2 [1 + Cos θ] u watts**

Efficiency, η = Output of the jet per second

 Input of the jet per second

 = Work done per second

 Energy supplied per sec,

 = ρa(V- u)2 [1 + Cosθ] u

 1 ρaV3

 2

 **η = 2 u (V- u)2 [1 + Cosθ]**

**V3**

A jet of water of diameter 75 mm strikes curved plate at its centre with a velocity of 23 m/s.The curved plate moving with a velocity of 8 m/s in the direction of jet. The jet is deflected through an angle of 1650. Assume the plate to be smooth. Find i) Force exerted on the plate in the direction of the jet. ii) Power of the jet iii) Efficiency of the jet.

Given data,

Diameter of the jet, d = 75 mm = 0.075 m

Absolute velocity of the jet, V = 23 m/s

Angle of deflection of the jet, (180-θ) = 1650, θ = 180 - 165 = 150

Velocity of the plate, u = 8 m/s

1. Force exerted by the jet on the plate in the direction of jet,

Fx = ρa(V- u)2 [1 + Cosθ]

 = 1000 x Π/4 x (0.075)2 x (23 - 8)2 (1 + cos15)

 **= 1954.17 N**

1. Workdone by the jet on the plate per second,

W = Fx x u = 1954.17 x 8 = **15633.36 Nm/s**

Power of the jet, P = Workdone by the jet per second

 = 15633.36 Watts

 **= 15.63 kW**

1. Efficiency of the jet, η = Work done per second

Energy supplied per sec

 = 15633.6

1 x 1000 x Π x (0.075)2 x 232

2 4

  **η = 0.5817 = 58.17 %**

F**ORCE EXERTED BY THE JET ON A MOVING CURVED PLATE AT ONE END TANGENTIALLY**

Consider a jet of fluid strikes on a curved plate tangentially at one of its tips as shown in fig. As the jet strikes tangentially, the loss of energy due to impact of jet will be zero.



|  |  |
| --- | --- |
| 1 | FORCE EXERTED BY THE JET ON A STATIONARY PLATE |
| a | When flat plate is held normal to the jet |  **Fx = ρaV2** |  |
| b | When flat plate is held incclined to the jet | **Fn = ρaV2 sin θ****Fx = Fn Sin θ** **= ρaV2 sin2θ****Fy = Fn Cos θ** **=ρaV2 Sin θ Cos θ** |  |
| c | When curved plate, jet striking at centre | **Fx = ρaV2 [1+ Cosθ ]****Fy = - ρaV2 Sinθ** |  |
| d | When curved plate, jet striking at one end tangentially |  |  |
| 2 | FORCE EXERTED BY THE JET ON A MOVING PLATE |
| a | When flat plate is held normal to the jet |  |  |
| b | When flat plate is held incclined to the jet |  |  |
| c | When curved plate, jet striking at centre |  |  |
| d | When curved plate, jet striking at one end tangentially |  |  |
| 3 | FORCE EXERTED BY THE JET ON A MOVING PLATE |  |  |