2.1 Tangent and Normal

1. Find the slope of the curve $y = x^3$ at (1, 1).

2. Find the slope of the curve $y = \tan x$ at $x = \frac{\pi}{3}$.

3. Find the slope of the curve $y = \tan x$ at $x = \frac{\pi}{4}$.

4. Find the slope of the curve $y = \log_e(\cos x)$ at $x = \frac{\pi}{4}$.

5. For what values of x will the tangent to the curve $y = 2x^3 - 9x^2 + 12x - 3$ be parallel to the X-axis?

6. At what points on the curve $y = x^4 - 4x^3 + 4x^2 + 4$ is the tangent parallel to the X-axis?

7. For what values of x will the tangent to the curve $y = \frac{x}{x^2+1}$ be parallel to the X-axis?

8. Find the values of x for which the tangent to the curve $y = \frac{x}{(1-x)^2}$ be parallel to the (i) X-axis, (ii) Y-axis?

9. Find the equation of the tangent and normal to the curve $y = 2\log x$ at the point (1, 0).

10. Find the equation of the tangent and normal to the semi circle $y = \sqrt{25 - x^2}$ at the point (4, 3).

11. Find the equation of the tangent and normal to the curve $y = \cos x$ at $x = \frac{\pi}{6}$.

12. Find the equation of the tangent to the curve $x = \cos 2t$ and $y = \sin 2t$ at $t = \frac{\pi}{8}$.

13. Find the equation of the tangent and normal to the rectangular hyperbola $x = \text{ct}, \text{y} = \frac{\text{c}}{\text{t}}$ at the point $\left(\text{ct}, \frac{\text{c}}{\text{t}}\right)$.

14. Find the equation of the tangent to the parabola $y^2 = 4ax$ at $(at^2, 2at)$.

15. Find the equation of the tangent to the parabola $y^2 = 4ax$ at $(a, 2a)$. Also find the slope of the normal at the above point.

16. Find the equation of the tangent and normal to the semi circle $x^2 + y^2 = 25$ at the point (3, -4).

2.2 Rate of Change of quantities

1. Find the rate of change of area of a circle w.r.t the radius.
2. Find the rate of change of volume of a cone with respect to its radius when the radius is equal to height.
3. Find the rate of change of volume of a sphere with respect to its radius.
4. Find the rate of change of volume of a cube with respect to its side x.
5. If $q = \frac{10p}{p-10}$, find the rate of change of q w.r.t p.

6. A circular patch of oil spreads out on water at the rate of 12 sq.cm/min. How fast is the radius increasing when the radius is 2 cm.
7. A circular plate contracts when cooled. Find the rate of decrease in area if the radius decreases at the rate 0.1 cm/min, when the radius is 4 cm.
8. A circular plate of radius 4 inches expands, when heated, at the rate of 0.25 inch/sec. Find how fast its area is increasing at the end of 6 sec.
9. A circular patch of oil spreads out on water, the area growing at the rate of 65 sq.cm/min. How fast is the radius increasing when the radius is 2 cm.
10. A stone is dropped into still water. The radius of the outer most ripple then formed increases at the rate of 6 cms per second. How fast is the area increasing when the radius is 16 cms?
11. A stone is thrown into still water causes concentric ripples. If the radius of the outer most ripple is increasing at the rate of 10cm/sec, how fast is the area of the disturbed water increasing when the outer ripple has a radius of 24 cms?
12. A balloon is spherical in shape. Gas is escaping from it at the rate of 10 cc/sec. How fast is the surface area shrinking when the radius is 15 cms?
13. A spherical balloon is inflated with air such that its volume increases at the rate 10 c.c per second. Find the rate at which its curved surface is increasing when its radius is 4 cm.
14. A spherical balloon is inflated with air such that its volume increases at the rate 5.c.c per second. Find the rate at which its curved surface is increasing when its radius is 7 cm.
15. A balloon always remains spherical, expands at the rate of 100 cc/sec. Find the rate at which radius is increasing when the radius is 3 cm.
16. Air is leaking out of a spherical balloon at the rate of 3 cubic inches per minute. When the radius is 5 inches, how fast is the radius decreasing?
17. A balloon is spherical in shape. Gas is escaping from it at the rate of 20 cc/sec. How fast is the surface area shrinking when the radius is 30 cm?
18. A spherical balloon is inflated with air such that its volume increases at the rate 7 c.c per second. Find the rate at which its curved surface is increasing when its radius is 14 cm.
19. A spherical rubber bladder of radius 3’’ has air pumped into it . If the radius increases at a uniform rate of 1’’ per minute, find the rate at which the volume is increasing at the end of 3 minutes.
20. The volume of a sphere is increasing at the rate of 20π cc/sec. Find the rate at which the surface area is increasing when the volume is $288\pi\ cm^3$.
21. Water is running out of a conical funnel at the rate of 1 cubic inch per second. If the radius of the funnel is 4 inches and altitude is 8 inches, find the rate at which the water level is dropping when its depth is 6 inches.
22. The sand falls into a conical pile at the rate of 10 cc/sec and the radius of the pile is always equal to half of its altitude. How fast is the altitude of the pile increasing, when the altitude is 150 cm?

2.3 Velocity and Acceleration

1. Find the velocity and acceleration at time t, of a particle moving according to the rule $s = 4t^2 - 3t$.
2. Find the velocity and acceleration of a particle moving under the law $s = 4t^3 + 5t^2 + 6t + 7$ at time t = 3, the units being kms and hour.
3. Find the velocity and acceleration at time t = 2 sec of a moving body whose displacement is given by $s = \frac{t^4}{2}$.
4. The displacement of a particle at time t is $s = \frac{1}{2}t^2 + \sqrt{t}$. Find the velocity at time t = 9 sec.
5. Find the velocity and acceleration at time t = 4 sec of a moving body whose displacement S meters is given by $s = \frac{1}{2}t^2 + \sqrt{t}$.
6. A particle moves in a straight line in such a way that its displacement at a time t seconds is given by $s = 9t^2 - t^3$. Find the displacement when the velocity is zero.
7. A particle is moving along a line according to the formula $s = t^3 - 9t^2 + 3t + 1$, where s is in meters and t in seconds. Find its acceleration when the velocity is 24 m/sec.
8. A particle moves in a vertical line as per the equation $s = 100t - 49t^2$, where s is in meters and t in seconds. What is its velocity at t = 1? At what time is its velocity zero? What is its acceleration at t = 1?
9. The distance S meters travelled by a particle in t seconds is given by $s = ae^t + be^{-t}$. Show that the acceleration of the body is always equal to the distance passed over.
10. The distance S meters travelled by a particle is given by $s = ae^{2t} + be^{-2t}$ where t represents the time. Prove that the acceleration varies as the distance.
11. If S denotes the displacement of a particle and $s = ae^{nt} + be^{-nt}$, show that the acceleration is proportional to S.
12. The displacement of a particle is given by $s = 4\cos 3t + 5\sin 3t$. Show that the acceleration is proportional to the displacement.
13. The displacement of a particle is given by $s = 5\cos nt + 4\sin nt$. Show that the acceleration is proportional to the displacement.

2.4 Increasing/Decreasing function

1. Prove that the function $x^3 + 6x^2 + 12x - 9$ is an increasing function for all values of x.
2. Prove that the function $x^3 - 6x^2 + 48x + 1$ is an increasing function for all values of x.
3. Prove that the function $x^3 - 3x^2 + 3x + 7$ is an increasing function for all values of x
4. Prove that the function $7 - 9x - 6x^2 - 2x^3$ is a decreasing function for all values of x.
5. Find the range of values of x for which the function $x^2 - 3x + 4$ is increasing or decreasing.
6. Find the range of values of x for which the function $x^3 - 6x^2 - 36x + 7$ is increasing or decreasing.

2.5 Maxima and Minima

1. Find the maximum value of y, if $y = 2x^3 - 9x^2 + 12x$.
2. Find the maximum value of the function $f(x) = 2 - 6x - x^2$.
3. Find the maximum and minimum values of the function $f(x) = x^3 - 6x^2 + 9x$.
4. Find the maximum and minimum values of $y = 4x^3 + 9x^2 - 12x + 2$.
5. A particle is projected vertically upwards and its height h feet at time t is given by $h = 60t - 16t^2$. Find the greatest height attained.
6. The deflection of a beam is given by $y = 4x^3 + 9x^2 - 12x + 2$ .Find the maximum deflection.
7. The deflection of a beam is given by $y = 2(100x - x^2)$ .Find the maximum deflection.
8. The bending moment of a rod 10 m long and weighing 40 kg and resting on supports at its ends at a distance x meters from one end is given by $M = 2(10x - x^2)$. Find the maximum bending moment.
9. An open box is to be made out of a square sheet of side 8 cm, by cutting off equal squares at each corner and turning up the sides. What size of squares should be cut in order that the volume of the box may be maximum?
10. An open box is to be made out of a square sheet of side 18 cm, by cutting off equal squares at each corner and turning up the sides. What size of squares should be cut in order that the volume of the box may be maximum?
11. The sum of the diameter and length of an open cylindrical vessel is 40 cm. Prove that the maximum volume is obtained when the radius is equal to the length.
12. A hollow vertical vessel to hold 100 cc of water is to be made so that the area of the metal used is minimum. Prove that the radius which will give minimum area is $\sqrt[3]{\frac{100}{\pi}}$ cms.
13. A cylindrical can open at one end is to have a volume of $64\ \text{cm}^3$. Find the radius and height such that the metal used is a minimum.
14. A big water tank is to be constructed with a square base, an open top and vertical walls, it is to contain 256 cubic meters of water. Find the sides of the base and the height if the sum of the area of the bottom and the walls is to be a minimum.
15. An open tank is to be constructed with a square base and vertical sides to hold a given quantity of water, show that the expense of lining it with lead will be the least if the depth is half the width.
16. Find the most economical dimensions for a cylindrical cup that holds a specified volume, if the cup has no top.
17. Find the maximum volume of a cone of slant height *l*.
18. Prove that a rectangle of fixed perimeter has its maximum area when it becomes a square.