Scoring Indicators

Course Name : Mathematics II

Course code : 2002

QID: 2106220097 **A**

Q No	Scoring Indicators	Split Score	Sub Total	Total Score
I	PART - A			9
I.1	$\begin{vmatrix} 3 & 4 \\ -2 & 3 \end{vmatrix} = 3\times 3 - {}^{-}2\times 4 = 9+8=17$	1		
I.2.	$A^T = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 4 & 1 \\ 5 & 2 & 3 \end{bmatrix}$	1		
I.3.	$\vec{a}+\vec{b} = 3\hat{i}+4\hat{j}-2\hat{k}+2\hat{i}-3\hat{j}-2\hat{k} = 5\hat{i}+\hat{j}-4\hat{k}$	1		
I.4.	Vector joining $(1,2,{}^{-}3)$ and $(3,1,1)$ $= (3-1)\hat{i}+(1-2)\hat{j}+(1-{}^{-}3)\hat{k} = 2\hat{i}-\hat{j}+4\hat{k}$	1		
I.5.	$\int \sec^2 x \cdot dx = \tan x + C$	1		
I.6.	$\int (2x+3)dx = 2\frac{x^2}{2}+3x+c = \underline{x^2+3x+C}$	1		
I.7.	$\int x^5 dx = \frac{x^6}{6}+C$	1		
I.8	Order - 2 , degree - 1	½+½		
I.9.	$A = \int_a^b y\, dx$	1		
II	PART - B			24
II.1.	$\begin{vmatrix} 3x & 7 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix}$			
	$\Rightarrow 3x\cdot 3 - 2\cdot 7 = 4\cdot 2 - 2\cdot 2$	1		
	$\Rightarrow 9x - 14 = 8-4$	1		
	$\Rightarrow 9x = 4+14 = 18$			
	$\Rightarrow x = \underline{\underline{2}}$	1		

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II.2	$A=\begin{bmatrix}3 & -2\\ -7 & 4\end{bmatrix} \Rightarrow 3A=\begin{bmatrix}9 & -6\\ -21 & 12\end{bmatrix}$	1		
	$B=\begin{bmatrix}2 & -3\\ 3 & 2\end{bmatrix} \Rightarrow 5B=\begin{bmatrix}10 & -15\\ 15 & 10\end{bmatrix}$	1	3	
	$3A-5B=\begin{bmatrix}9-10 & -6- -15\\ -21-15 & 12-10\end{bmatrix}=\begin{bmatrix}-1 & 9\\ -36 & 2\end{bmatrix}$	1		
II.3.	$A+B=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}+\begin{bmatrix}-1 & 2\\ 2 & 1\end{bmatrix}=\begin{bmatrix}2 & 4\\ 3 & 5\end{bmatrix}$	1		
	$(A+B)C=\begin{bmatrix}2 & 4\\ 3 & 5\end{bmatrix}\begin{bmatrix}3 & -2\\ 1 & 3\end{bmatrix}=\begin{bmatrix}2\times3+4\times1 & 2\times-2+4\times3\\ 3\times3+5\times1 & 3\times-2+5\times3\end{bmatrix}$	1	3	
	$=\begin{bmatrix}6+4 & -4+12\\ 9+5 & -6+15\end{bmatrix}=\begin{bmatrix}10 & 8\\ 14 & 9\end{bmatrix}$	1		
II.4.	$\vec{a}\cdot\vec{b}=(\hat{i}-2\hat{j}+3\hat{k})\cdot(3\hat{i}-2\hat{j}+\hat{k})$			
	$=1\times3-2\times-2+3\times1=3+4+3=10$	1		
	$\lvert\vec{a}\rvert=\sqrt{1^2+(-2)^2+3^2}=\sqrt{1+4+9}=\sqrt{14}$	½		
	$\lvert\vec{b}\rvert=\sqrt{3^2+(-2)^2+1^2}=\sqrt{9+4+1}=\sqrt{14}$	½		
	$\therefore\ \theta=\cos^{-1}\dfrac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}=\cos^{-1}\dfrac{10}{\sqrt{14}\sqrt{14}}=\cos^{-1}\left(\dfrac{10}{14}\right)$	1		
II.5.	$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -2\\ 1 & -3 & 4\end{vmatrix}=\hat{i}\begin{vmatrix}1 & -2\\ -3 & 4\end{vmatrix}-\hat{j}\begin{vmatrix}3 & -2\\ 1 & 4\end{vmatrix}+\hat{k}\begin{vmatrix}3 & 1\\ 1 & -3\end{vmatrix}$	1+1	3	
	$=(4-6)\hat{i}-\hat{j}(12+2)+\hat{k}(-9-1)$			
	$=-2\hat{i}-14\hat{j}-10\hat{k}$	1		
II.6.	$\int\sin2x\cdot\cos3x\cdot dx=\int\frac{1}{2}\left[\sin(2x+3x)+\sin(2x-3x)\right]dx$	1		
	$=\frac{1}{2}\int\left[\sin5x-\sin x\right]dx=\frac{1}{2}\left[\frac{-\cos5x}{5}-\cos x\right]+c$	1+1		
II.7.	$\int\tan x\cdot dx=\int\frac{\sin x}{\cos x}\cdot dx$	1		
	Put $u=\cos x \Rightarrow du=-\sin x\,dx$	1	3	
	$\int\tan x\,dx=\int\frac{\sin x}{\cos x}dx=\int\frac{-du}{u}=-\log u=\log u^{-1}=\log\frac{1}{u}=\log\sec x+c$	1		

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II.8.	$\int \frac{2+\sin x}{\cos^2 x} dx = \int\left(\frac{2}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right) dx$	1		
	$= \int\left(\frac{2}{\cos^2 x} + \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\right) \cdot dx$	1	3	
	$= \int(2\sec^2 x + \tan x \cdot \sec x) \cdot dx$	1		
	$= 2\tan x + \sec x + C$	1		
II.9.	$\text{Area} = A = \int_a^b y\,dx = \int_0^{\pi/2}(x+\sin x)dx$	1		
	$= \left[\frac{x^2}{2} - \cos x\right]_0^{\pi/2} = \left[\frac{\pi^2/4}{2} - \cos\frac{\pi}{2} - (0-\cos 0)\right]$	1	3	
	$= \left(\frac{\pi^2}{8} - 0\right) - (0-1) = \frac{\pi^2}{8} + 1$	1		
II.10. Answer is in Page 10.				
	PART C			42
III.a.	$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{vmatrix} = 1\begin{vmatrix} 3 & 1 \\ -1 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$			
	$= 1(-3+1) - 1(-2-1) + 1(-2-3)$			
	$= 1 \times -2 - 1 \times -3 + 1 \times -5 = -2+3-5 = -4$	1		
	$\Delta_1 = \begin{vmatrix} 3 & 1 & 1 \\ 6 & 3 & 1 \\ -3 & -1 & -1 \end{vmatrix} = 3(-3+1) - 1(-6+3) + 1(-6+9)$			
	$= 3 \times -2 - 1 \times -3 + 1 \times 3 = -6+3+3 = 0$	1		
	$\Delta_2 = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 6 & 1 \\ 1 & -3 & -1 \end{vmatrix} = 1(-6+3) - 3(-2-1) + 1(-6-6)$			
	$= 1 \times -3 - 3 \times -3 + 1 \times -12 = -3+9-12 = -6$	1	7	
	$\Delta_3 = \begin{vmatrix} 1 & 1 & 3 \\ 2 & 3 & 6 \\ 1 & -1 & -3 \end{vmatrix} = 1(-9+6) - 1(-6-6) + 3(-2-3)$			
	$= 1 \times -3 - 1 \times -12 + 3 \times -5$			
	$= -3+12-15 = -18+12 = -6$	1		
	$x = \frac{\Delta_1}{\Delta} = \frac{0}{-4} = 0, \quad y = \frac{\Delta_2}{\Delta} = \frac{-6}{-4} = \frac{3}{2}, \quad z = \frac{\Delta_3}{\Delta} = \frac{-6}{-4} = \frac{3}{2}$	1+1+1		

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IV	$5x-y=3$			
	$4x+2y=-1$			
	Matrix form $AX=B$			
	$\begin{bmatrix}5 & -1\\4 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3\\-1\end{bmatrix}$	1		
	$C_{11}=2,\ C_{12}=-4,\ C_{21}=1,\ C_{22}=5$	1		
	Cofactor matrix $C=\begin{bmatrix}2 & -4\\1 & 5\end{bmatrix}$			
	$Adj\ A=\begin{bmatrix}2 & 1\\-4 & 5\end{bmatrix}$	1		
	$\lvert A\rvert=\begin{vmatrix}5 & -1\\4 & 2\end{vmatrix}=10+4=14$	1	7	
	$A^{-1}=\frac{Adj\ A}{\lvert A\rvert}=\frac{1}{14}\begin{bmatrix}2 & 1\\-4 & 5\end{bmatrix}$	1		
	$\therefore$ Sln is $X=A^{-1}B=\frac{1}{14}\begin{bmatrix}2 & 1\\-4 & 5\end{bmatrix}\begin{bmatrix}3\\-1\end{bmatrix}$			
	$=\frac{1}{14}\begin{bmatrix}2\times3+1\times-1\\-4\times3+5\times-1\end{bmatrix}=\frac{1}{14}\begin{bmatrix}6-1\\-12-5\end{bmatrix}$			
	$=\frac{1}{14}\begin{bmatrix}5\\-17\end{bmatrix}=\begin{bmatrix}5/14\\-17/14\end{bmatrix}$	2		
	$x=\frac{5}{14},\ y=\frac{-17}{14}$			
V a.	$\vec{OA}=\hat{i}+3\hat{j}+2\hat{k}$			
	$\vec{OB}=2\hat{i}-\hat{j}+\hat{k}$	1		
	$\vec{OC}=2\hat{i}-3\hat{j}+4\hat{k}$			
	$\vec{AB}=\vec{OB}-\vec{OA}=(2\hat{i}-\hat{j}+\hat{k})-(\hat{i}+3\hat{j}+2\hat{k})$	1		
	$=\hat{i}-4\hat{j}-\hat{k}$		4	
	$\vec{AC}=\vec{OC}-\vec{OA}=(2\hat{i}-3\hat{j}+4\hat{k})-(\hat{i}+3\hat{j}+2\hat{k})$			
	$=\hat{i}-6\hat{j}+2\hat{k}$	1		
	$\vec{AB}\times\vec{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\1 & -4 & -1\\1 & -6 & 2\end{vmatrix}=\hat{i}(-8-6)-\hat{j}(2+1)+\hat{k}(-6+4)$			
	$=-14\hat{i}-3\hat{j}-2\hat{k}$	1		

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V. b	$\vec{F} = 5\hat{i} + 4\hat{j} + \hat{k}$			
	displacement vector $= 3\hat{i} + 2\hat{j} - 5\hat{k}$.			
	Work done $= \vec{F} \cdot \vec{AB} = (5\hat{i} + 4\hat{j} + \hat{k}) \cdot (3\hat{i} + 2\hat{j} - 5\hat{k})$	1+1	3	7
	$= 5 \times 3 + 4 \times 2 + 1 \times {}^{-}5 = 15 + 8 - 5$			
	$= \underline{18 \text{ units}}$	1		
VI. a.	A vector ⊥re to $\vec{a}$ & $\vec{b} = \vec{a} \times \vec{b}$	1		
	$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 6 & -3 & 2 \end{vmatrix} = \hat{i}(4+3) - \hat{j}(4-6) + \hat{k}(-6-12)$ $= 7\hat{i} + 2\hat{j} - 18\hat{k}$	2		
	unit vector in the direction of $\vec{a} \times \vec{b}$ is		5	
	$= \dfrac{\vec{a} \times \vec{b}}{\lvert \vec{a} \times \vec{b} \rvert} = \dfrac{7\hat{i} + 2\hat{j} - 18\hat{k}}{\sqrt{7^2 + 2^2 + (18)^2}}$	1		
	$= \dfrac{7\hat{i} + 2\hat{j} - 18\hat{k}}{\sqrt{49 + 4 + 324}} = \dfrac{7\hat{i} + 2\hat{j} - 18\hat{k}}{\sqrt{377}}$	1		
b.	$\vec{a}$ and $\vec{b}$ are ⊥re iff $\vec{a} \cdot \vec{b} = 0$	1		7
	$\vec{a} \cdot \vec{b} = (3\hat{i} - \hat{j} + 2\hat{k}) \cdot (2\hat{i} + 2\hat{j} - 2\hat{k})$		2	
	$= 3 \times 2 - 1 \times 2 + 2 \times {}^{-}2 = 6 - 2 - 4 = 0$	1		
	∴ $\vec{a}$ and $\vec{b}$ are ⊥re to each other.			
VII a.	Since $\vec{a} = p\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{j} - 4\hat{k}$			
	are ⊥re $\vec{a} \cdot \vec{b} = 0$	1		
	ie, $\vec{a} \cdot \vec{b} = p \cdot {}^{-}1 + 2 \cdot 3 + 3 \cdot {}^{-}4 = 0$	1		
	$\Rightarrow -p + 6 - 12 = 0$		3	
	$\Rightarrow {}^{-}p - 6 = 0$			
	$\Rightarrow \underline{p = +6}$	1		

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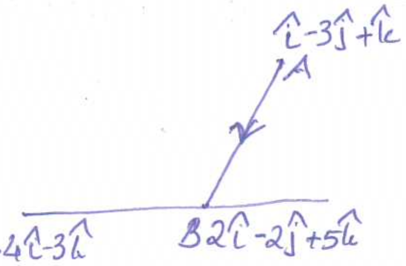
VII b.	$\vec{a}+\vec{b} = 2\hat{i}+2\hat{j}-5\hat{k}+2\hat{i}+\hat{j}+3\hat{k}$			
	$= 4\hat{i}+3\hat{j}-2\hat{k}$	1		
	unit vector $= \dfrac{\vec{a}+\vec{b}}{\lvert\vec{a}+\vec{b}\rvert}$	1		
	$= \dfrac{4\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{4^2+3^2+(-2)^2}} = \dfrac{4\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{16+9+4}}$	1	4	7
	$= \dfrac{4\hat{i}+3\hat{j}-2k}{\sqrt{29}}$	1		
VIII a.				
	$\vec{r} = \vec{AB} = \vec{OB}-\vec{OA}$			
	$= (2\hat{i}-2\hat{j}+5\hat{k})-(\hat{i}-3\hat{j}+\hat{k})$	1		
	$= \hat{i}+\hat{j}+4\hat{k}$			
	Moment vector $= \vec{r}\times\vec{F} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 4\\ 4 & 0 & -3\end{vmatrix}$			
	$= \hat{i}(-3-0)-\hat{j}(-3-16)+\hat{k}(0-4)$	1+2	5	
	$= -3\hat{i}+19\hat{j}-4\hat{k}$			
	$\therefore$ Moment $= \lvert$moment vector$\rvert = \sqrt{(-3)^2+19^2+(-4)^2}$			
	$= \sqrt{386}$	1		
b.	$\vec{AB} = \vec{OB}-\vec{OA}$			7
	$\vec{OA} = 2\hat{i}-\hat{j}-\hat{k}$, $\vec{OB} = -\hat{i}-3\hat{j}+3\hat{k}$	1		
	$\vec{AB} = \vec{OB}-\vec{OA} = (-\hat{i}-3\hat{j}+3\hat{k})-(2\hat{i}-\hat{j}-\hat{k})$		2	
	$= -3\hat{i}-2\hat{j}+4\hat{k}$	1		

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IX a.	$\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$			
	Put $\tan x = u \Rightarrow du = \sec^2 x\,dx$	1		
	$\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx = \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + c$	1+1	4	
	$= \sin^{-1}(\tan x) + c$	1		
IX. b	$\int (x+1)(x+2)\,dx = \int (x^2+3x+2)\,dx$	1		7
	$= \left[\frac{x^3}{3} + 3\frac{x^2}{2} + 2x\right] + c$	2	3	
X. a.	$\int_0^4 x\sqrt{x^2+9}\,dx.$			
	Put $x^2+9 = u \Rightarrow 2x\,dx = du \Rightarrow x\,dx = \frac{du}{2}$	1		
	when $x=0 \Rightarrow u = 9$			
	$x=4 \Rightarrow u = 25$	1		
	$\int_0^4 x\sqrt{x^2+9}\,dx = \int_9^{25} \sqrt{u}\cdot\frac{du}{2} = \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right]_9^{25}$	1	4	
	$= \frac{1}{3}\left[25^{3/2} - 9^{3/2}\right] = \frac{1}{3}\left[125-27\right]$			
	$= \frac{1}{3}[98] = \frac{98}{3}$	1		
X. b.	$\int x^2 \cdot \log x \cdot dx$			7
	$\int uv\cdot dx = u\cdot\int v\,dx - \int\left[\left(\frac{du}{dx}\right)\cdot\int v\,dx\right]dx$	1		
	$= \log x \cdot \int x^2\,dx - \int\left(\frac{1}{x}\cdot\int x^2\,dx\right)$			
	$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x}\cdot\frac{x^3}{3}\,dx$			
	$= \frac{x^3}{3}\log x - \frac{1}{3}\int x^2\,dx$	1	3	
	$= \frac{x^3}{3}\log x - \frac{1}{3}\cdot\frac{x^3}{3}$			
	$= \frac{x^3}{3}\log x - \frac{x^3}{9} + c$	1		

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XI. a.	$\int_1^2 \log x \cdot dx$.			
	$\int \log x \, dx = \int \log x \cdot 1 \cdot dx$			
	$\int uv\,dx = u \cdot \int v\,dx - \int \left[\frac{du}{dx} \cdot \int v\,dx\right] dx$.	1		
	$\int \log x \cdot 1\,dx = \log x \cdot \int 1\,dx - \int \left(\frac{1}{x} \cdot \int 1\,dx\right) dx$	1		
	$= (\log x) \cdot x - \int \frac{1}{x} \cdot x \cdot dx$			
	$= x \log x - \int 1\,dx$		4	
	$= x \log x - x$	1		
	$\int_1^2 \log x \cdot dx = \left[x \log x - x\right]_1^2 = \left[(2\log 2 - 2) - (1 \log 1 - 1)\right]$			
	$= [2\log 2 - 2 + 1]$ $= 2\log 2 - 1$	1		7
b.	$\int \cos 4x \cdot \sin 2x \cdot dx$			
	$= \frac{1}{2} \int [\sin(4x+2x) - \sin(4x-2x)]\,dx$	1		
	$= \frac{1}{2} \int (\sin 6x - \sin 2x)\,dx$	1	3	
	$= \frac{1}{2}\left[\frac{-\cos 6x}{6} + \frac{\cos 2x}{2}\right] + C$	1		
XII. a.	$\int \cos^2 2x\,dx = \int \frac{1+\cos 4x}{2}\,dx$	1		
	$= \frac{1}{2} \int (1 + \cos 4x)\,dx$	1	3	
	$= \frac{1}{2}\left[x + \frac{\sin 4x}{4}\right] + C$	1		
XII b.	$\int_1^2 \frac{2x+1}{x^2+x+1} \cdot dx$ Put $x^2+x+1 = u$, $(2x+1)\,dx = du$	1		7
	when $x=1 \Rightarrow u = 3$, $x = 2 \Rightarrow u = 7$	1	4	
	$= \int_3^7 \frac{du}{u} = [\log u]_3^7 = \log 7 - \log 3$			
	$= \log 7/3$	1+1		

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XIII a.	$A=\int_a^b y\,dx = \int_0^4 \sqrt{4x}\,dx$	1		
	$=2\int_0^4 \sqrt{x}\,dx = 2\cdot\left[\frac{x^{3/2}}{3/2}\right]_0^4$	1	3	
	$=\frac{4}{3}\left[4^{3/2}-0\right]=\frac{4}{3}\times 8=\frac{32}{3}$ sq.unit	1		7
XIII b.	$dy=e^{3x+y}dx \Rightarrow dy=e^{3x}e^{y}dx$	1		
	$\Rightarrow \frac{dy}{e^{y}}=e^{3x}dx$			
	$\Rightarrow e^{-y}dy=e^{3x}dx$	1	4	
	$\Rightarrow \frac{e^{-y}}{-1}=\frac{e^{3x}}{3}+C$	1		
	$\Rightarrow -e^{-y}=\frac{e^{3x}}{3}+C$	1		
XIV a.	$\frac{dy}{dx}+2y\tan x=\sin x.$			
	It is a linear d.e. of the form $\frac{dy}{dx}+Py=Q.$			
	$P=2\tan x,\ Q=\sin x.$	1		
	$I.F=e^{\int P\,dx}=e^{\int 2\tan x\,dx}=e^{2\log\sec x}$			
	$=e^{\log\sec^2 x}=\sec^2 x$	1		
	$\therefore$ sln is $y\,e^{\int P\,dx}=\int Q\,e^{\int P\,dx}\,dx$	1		
	$\Rightarrow y\cdot\sec^2 x=\int \sin x\cdot\sec^2 x\cdot dx$	1	5	
	$\Rightarrow y\sec^2 x=\int \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\,dx$			
	$\Rightarrow y\sec^2 x=\int \sec x\cdot\tan x\cdot dx$			
	$\Rightarrow y\sec^2 x=\sec x+C$	1		

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XIV b	$\frac{dy}{dx} = 2$ $\Rightarrow dy = 2dx$ entegrating $\Rightarrow y = 2x + c$	 1 1	 2	
II 10.	$\frac{dy}{dx} = 2y$ $\Rightarrow \frac{dy}{y} = 2dx$ $\Rightarrow \log y = 2x + c$ $\Rightarrow y = ke^{2x}$	 1 1 1	 3	