

1.1 Limits and Continuity

1.  $\lim_{x \rightarrow 3} \frac{5x+1}{x+1}$

2.  $\lim_{x \rightarrow 3} \frac{x^2 - 3x}{x^2 - 9}$

3.  $\lim_{x \rightarrow 0} \frac{2x-3}{3x+4}$

4.  $\lim_{x \rightarrow 1} \frac{x^2 + 4x - 5}{x^2 + x - 2}$

5.  $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 + x - 6}$

6.  $\lim_{x \rightarrow 4} \frac{x-4}{x^2 - 16}$

7.  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$

8.  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$

9.  $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$

10.  $\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16}$

11.  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$

12.  $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$

13.  $\lim_{\theta \rightarrow 0} \frac{\sin 5\theta}{6\theta}$

14.  $\lim_{\theta \rightarrow 0} \frac{\sin \frac{\theta}{2}}{\theta}$

15.  $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

16.  $\lim_{\theta \rightarrow 0} \frac{\sin 4\theta + \sin 2\theta}{6\theta}$

17.  $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$

18.  $\lim_{\theta \rightarrow 0} \frac{\sin 3\theta \cdot \cos \theta}{\theta}$

19.  $\lim_{x \rightarrow \infty} \frac{2x+3}{x+1}$

20.  $\lim_{x \rightarrow \infty} \frac{1-2x}{1+2x}$

21.  $\lim_{x \rightarrow \infty} \frac{x^2 + 2x + 3}{2x^2 - 4x + 1}$

22.  $\lim_{x \rightarrow \infty} \frac{x^2 - 2x + 3}{2x^2 - 3x}$

23.  $\lim_{x \rightarrow \infty} \frac{x^3 - 3x + 5}{2x^3 - 4x - 6}$

24.  $\lim_{x \rightarrow \infty} \frac{x^2 - 2x + 8}{4x^3 - 3}$

25. Examine whether the function given by  $f(x) = \begin{cases} x^2 + 3 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$  is continuous or not at  $x = 0$ .

26. Find the value of  $a$  if  $f(x) = \begin{cases} \sin x & \text{if } x \neq 0 \\ a & \text{if } x = 0 \end{cases}$  is continuous at  $x = 0$ .

27. Find the value of  $k$  if  $f(x) = \begin{cases} x^2 - 4 & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$  is continuous at  $x = 2$ .

28. Examine whether the function given by  $f(x) = \begin{cases} x^3 - 8 & \text{if } x \neq 2 \\ 12 & \text{if } x = 2 \end{cases}$  is continuous or not at  $x = 2$ .

29. Find the value of  $k$  if  $f(x) = \begin{cases} k \cos x & \text{if } x \neq \frac{\pi}{2} \\ \frac{\pi - 2x}{3} & \text{if } x = \frac{\pi}{2} \end{cases}$  is continuous at  $x = \frac{\pi}{2}$ .

30. Find the value of  $\lambda$  if  $f(x) = \begin{cases} \lambda x + 1 & \text{if } x = \pi \\ \cos x & \text{if } x \neq \pi \end{cases}$  is continuous at  $x = \pi$ .

31. Find the value of  $k$  if  $f(x) = \begin{cases} kx^2 & \text{if } x \neq 2 \\ 3 & \text{if } x = 2 \end{cases}$  is continuous at  $x = 2$ .

**Differentiation**

**1.2 Differentiation by basic Rules**

- Find  $\frac{dy}{dx}$ , if  $y = 5 \tan x - x^4$ .
- Find  $\frac{dy}{dx}$ , if  $y = x + \sin x + 5 \tan x$ .
- Find  $\frac{dy}{dx}$ , if  $y = 2\sqrt{x} - \frac{3}{x} - \frac{1}{\sqrt{x}}$ .
- Find  $\frac{dy}{dx}$ , if  $y = 4e^x - 3 \operatorname{cosec} x$ .
- Find  $\frac{dy}{dx}$ , if  $y = 3x - 2\sqrt{x} + 5 \sec x$ .
- Find  $\frac{dy}{dx}$ , if  $y = \frac{8}{x^2} - 2 \cot x$ .

**1.3 Differentiation by Product and Quotient Rules**

- Find  $\frac{dy}{dx}$ , if  $y = x^2 e^x$ .
- Find  $\frac{dy}{dx}$ , if  $y = x^3 \log x$ .
- Find  $\frac{dy}{dx}$ , if  $y = x^n \log x$ .
- Find  $\frac{dy}{dx}$ , if  $y = x^3 \cos x$ .
- Find the derivative of:  $(1 + x^2) \tan^{-1} x$ .
- Find  $\frac{dy}{dx}$ , if  $y = x^2 \sin^{-1} x$ .
- Find  $\frac{dy}{dx}$ , if  $y = \frac{1 - x^2}{1 + x^2}$ .
- Find  $\frac{dy}{dx}$ , if  $y = \frac{e^x - 1}{e^x + 1}$ .
- Find the derivative of:  $\frac{\sin x}{x}$ .
- Find  $\frac{dy}{dx}$ , if  $y = \frac{\sec x}{1 + x}$ .
- Find  $\frac{dy}{dx}$ , if  $y = \frac{\sin x}{1 + 2x}$ .
- Differentiate w. r. t x:  $\frac{\sin x}{x^2}$ .
- Find  $\frac{dy}{dx}$ , if  $y = \frac{x^2}{\tan x}$ .
- Find  $\frac{dy}{dx}$ , if  $y = \frac{\sin^{-1} x}{1 - x^2}$ .
- Find  $\frac{dy}{dx}$ , if  $y = \frac{x \sin x}{1 + x^2}$ .
- If  $y = \frac{x \tan^{-1} x}{1 + x^2}$ , find  $\frac{dy}{dx}$ .
- Find  $\frac{dy}{dx}$ , if  $y = \frac{e^x \sin x}{1 + \log x}$ .

**1.4 Differentiation by Chain Rule**

- Find  $\frac{dy}{dx}$ , if  $y = (2x^2 + 5x + 7)^{12}$ .
- Find  $\frac{dy}{dx}$ , if  $y = (1 - 2x + 7x^2)^{10}$ .
- Differentiate w. r. t x:  $\tan^2 3x$ .
- Differentiate  $\sin^2 x$  w. r. t x.
- Differentiate w. r. t x:  $\sin^2 5x$ .
- Find  $\frac{dy}{dx}$ , if  $y = 2 \sec^2(5x + 1)$ .
- Find  $\frac{dy}{dx}$ , if  $y = \sec^2(e^x)$ .
- Find  $\frac{dy}{dx}$ , if  $y = \tan \sqrt{x}$ .

9. Find  $\frac{dy}{dx}$ , if  $y = \frac{1}{\sec \sqrt{x}}$ .
10. Find  $\frac{dy}{dx}$ , if  $y = \sin^{-1}(2x)$ .
11. Find the derivative of  $y = (e^x + e^{-x})^2$  w.r.t x.
12. Differentiate w. r. t x:  $\log(\sin x)$ .
13. Find  $\frac{dy}{dx}$ , if  $y = \log \sin \sqrt{x}$ .
14. Find  $\frac{dy}{dx}$ , if  $y = \log(x + \sqrt{1+x^2})$ .
15. Find  $\frac{dy}{dx}$ , if  $y = \log(\cos ecx - \cot x)$ .
16. Find  $\frac{dy}{dx}$ , if  $y = \log(\sec x + \tan x)$ .
17. Find  $\frac{dy}{dx}$ , if  $y = \log(2x + \sin x)$ .
18. Find the derivative of :  $e^{2x} \log x$ .
19. Differentiate w. r. t x:  $x^2 \sin^5 x$ .
20. Differentiate w. r. t x:  $e^{2x} \cos 3x$ .
21. Differentiate w. r. t x:  $\frac{\sin(\log x)}{x}$ .
22. Find  $\frac{dy}{dx}$ , if  $y = \frac{1 + \cos x}{(x + \sin x)^3}$ .
23. Find  $\frac{dy}{dx}$ , if  $y = \frac{\tan^{-1} x}{(1+x^2)^2}$ .
24. Differentiate w. r. t x:  $\frac{\sin 2x}{1 + \cos 2x}$ .
25. Find  $\frac{dy}{dx}$ , if  $y = \frac{e^{2x} \log x}{x^2}$ .

### 1.5 Differentiation of Implicit forms

1. If  $2x^2 + 3xy + 5y^2 = 0$  find  $\frac{dy}{dx}$ .
2. Find  $\frac{dy}{dx}$ , if  $x^2 + xy + y^2 = 0$ .
3. Find  $\frac{dy}{dx}$ , if  $x^2 - xy^2 + 1 = 0$ .
4. Find  $\frac{dy}{dx}$ , if  $x^2 y^2 + y = x$ .
5. Find  $\frac{dy}{dx}$ , if  $x^3 + y^3 = 3xy$ .
6. Find  $\frac{dy}{dx}$ , if  $x^2 + 2xy + y^2 = 0$ .
7. Find  $\frac{dy}{dx}$ , if  $x^2 y^2 = x^3 + y^3 + 3xy$ .
8. Find  $\frac{dy}{dx}$ , if  $x^3 y - x^2 + xy - 1 = 0$ .
9. Find  $\frac{dy}{dx}$ , if  $ax^2 + 2hxy + by^2 = 0$ .
10. If  $x^2 + y^2 = 2axy$  find  $\frac{dy}{dx}$ .
11. Find  $\frac{dy}{dx}$ , if  $x^3 + y^3 = 3axy$ .
12. If  $x^{2/3} + y^{2/3} = a^{2/3}$  find  $\frac{dy}{dx}$ .
13. If  $y \log x = x - y$ , prove that  $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$ .

### 1.6 Differentiation of Parametric forms

1. If  $x = at^2$ ,  $y = 2at$ , find  $\frac{dy}{dx}$ .
2. Find  $\frac{dy}{dx}$ , if  $x = a \cos \theta$ ,  $y = a \sin \theta$ .
3. Differentiate :  $x = a \sec \theta$ ,  $y = b \tan \theta$ .
4. Find  $\frac{dy}{dx}$ , if  $x = a \cos ec \theta$ ,  $y = b \cot \theta$ .

5. Find  $\frac{dy}{dx}$ , if  $x = 2 \sin t$ ,  $y = \cos 2t$ .
6. Find  $\frac{dy}{dx}$ , if  $x = 2 \sin \theta$ ,  $y = \sin 2\theta$ .
7. Find  $\frac{dy}{dx}$ , if  $x = -\sin \theta$ ,  $y = \cos^2 \theta$ .
8. Find  $\frac{dy}{dx}$ , if  $x = a \cos^3 \theta$ ,  $y = b \sin^3 \theta$ .
9. Find  $\frac{dy}{dx}$ , if  $x = a \cot \theta$ ,  $y = b \operatorname{cosec}^2 \theta$ .
10. Find  $\frac{dy}{dx}$ , if  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$ .
11. If  $x = a(\theta + \sin \theta)$ ,  $y = a(1 - \cos \theta)$ , prove that  $\frac{dy}{dx} = \tan\left(\frac{\theta}{2}\right)$ .
12. Find  $\frac{dy}{dx}$ , if  $x = 3 \sin \theta - \sin^3 \theta$ ,  $y = 3 \cos \theta - \cos^3 \theta$ .

### Second order derivatives

1. Find the second derivative of (i)  $\frac{x}{x-2}$  (ii)  $\frac{x^2}{x+3}$  (iii)  $\frac{x}{4} + \frac{4}{x}$  (iv)  $y = e^{3x}$
2. If  $p = \frac{x}{x+R}$ , find  $\frac{d^2 p}{dx^2}$ .
3. Find  $\frac{d^2 y}{dx^2}$  if  $y = x^2 + \frac{2}{x}$ .
4. Find  $\frac{d^2 y}{dx^2}$  if  $y = \sin x$ .
5. If  $y = a \sin x + b \cos x$  show that  $\frac{d^2 y}{dx^2} + y = 0$ .
6. If  $y = e^x + e^{-x}$  show that  $\frac{d^2 y}{dx^2} = y$ .
7. If  $y = \cos x$ , prove that  $\frac{d^2 y}{dx^2} + y = 0$ .
8. If  $y = \sin x \cos x$  show that  $\frac{d^2 y}{dx^2} + 4y = 0$ .
9. If  $y = e^{ax}$ , prove that  $\frac{d^2 y}{dx^2} - a \frac{dy}{dx} = 0$ .
10. If  $xy = c^2$ , prove that  $xy_2 + 2y_1 = 0$ .
11. If  $y = \sin^{-1} x$ , prove that  $(1-x^2)y'' - xy' = 0$ .
12. If  $y = x \sin x$ , prove that  $\frac{d^2 y}{dx^2} + y = 2 \cos x$ .
13. If  $y = x \cos x$ , prove that  $\frac{d^2 y}{dx^2} + y + 2 \sin x = 0$ .
14. If  $y = a e^x + b e^{2x}$ , prove that  $\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = 0$ .
15. If  $y = a \cos(\log x) + b \sin(\log x)$ , prove that  $x^2 y'' + xy' + y = 0$ .
16. If  $y = x^2 \sin x$ , prove that  $x^2 y'' - 4xy' + (x^2 + 6)y = 0$ .
17. If  $y = x^2 \cos x$ , prove that  $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + (x^2 + 6)y = 0$ .
18. If  $xy = ax^2 + \frac{b}{x}$ , prove that  $x^2 y'' - 2xy' - 2y = 0$ .
19. If  $y = a \cos px + b \sin px$ , where a, p and b are constants, show that  $\frac{d^2 y}{dx^2}$  is proportional to y.