REVIEW OF TOS-I

FORCE :- It is defined as the which changes or - tens to change the states of rest of uniform motion of a body along a - straight line.

REACTION:- When a number of forces acting on a body is supported on another body then the second body exerts a force known as reaction.

Types of support

Simple Support 

Roller Support 

Pin joint 

Smooth Surface support 

Fixed support 

Types of support beams

cantilever beams

Simple beams

over hanging beam

continous beam

CENTROID

$$\bar{x} = \frac{\int x\,dA}{\int dA}$$

$$\bar{y} = \frac{\int y\,dA}{\int dA}$$

Centroid from compound figure

$$\bar{x} = \frac{A_1x_1 + A_2x_2 \cdots}{A_1 + A_2 \cdots}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 \cdots}{A_1 + A_2 \cdots}$$

Moment of Inertia

$$I_x = \int y^2\,dA$$

$$I_y = \int x^2\,dA$$

parallel axis MI $= I_{AB} + Ah^2$

$\perp^r$ axis $I_{zz} = I_{xx} + I_{yy}$

Bending stress $\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$

Shear stress in beams $\tau = \frac{FA\bar{y}}{Ib}$

$\tau \rightarrow$ shear stress

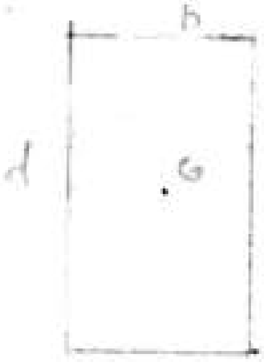
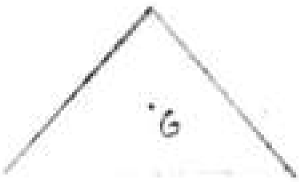
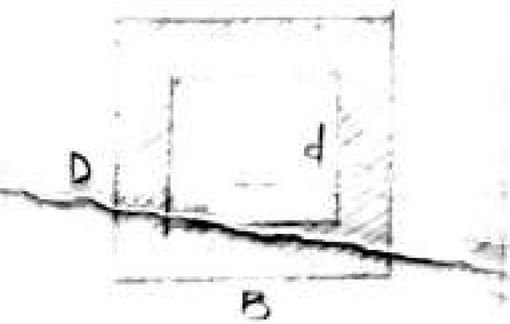
$F \rightarrow$ shear force

$A \rightarrow$ Area above neutral axis

$I \rightarrow$ Moment of inertia

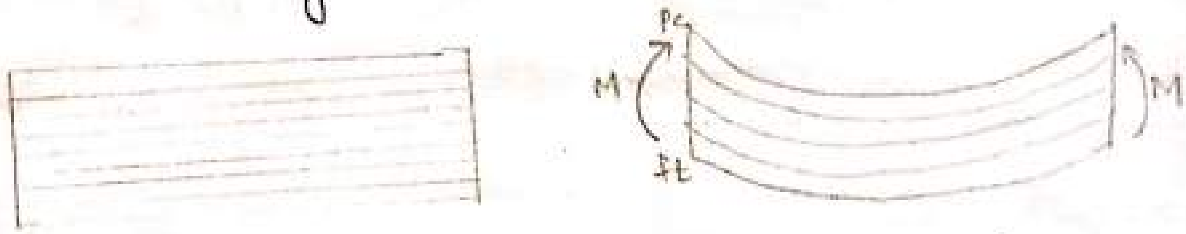
$b \rightarrow$

$\bar{y} \rightarrow$ centre of gravity

Figure	Centroid	MI
		$I_{xx} = \frac{bd^3}{12}$
		$\frac{bd^3}{3}$
		$\frac{bh^3}{12}$
		$\frac{bh^3}{36}$
		$\frac{BD^3}{12} - \frac{bd^3}{12}$

Bending Stress

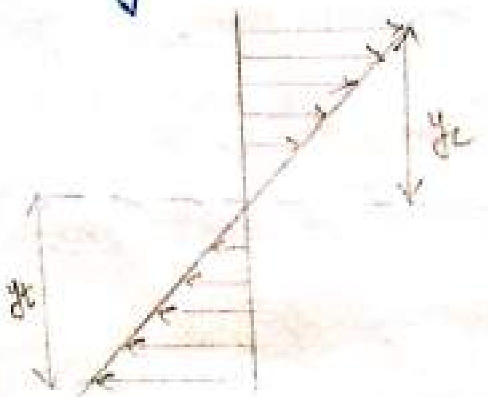
When a beam is subjected to external load. the beam has a tentency to bent internal stresses are develop in the beam to resist this bending stress are called bending stresses.



Fc → Bending compressive stresses

Ft → Bending tensile stresses.

Bending stress diagram

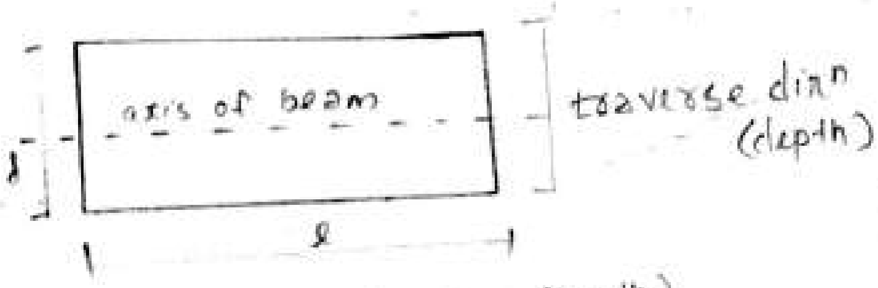


$f = \frac{E}{R} y$ → Distance from Na

$\tau = \frac{FA\bar{y}}{Ib}$ → Shearing stress.

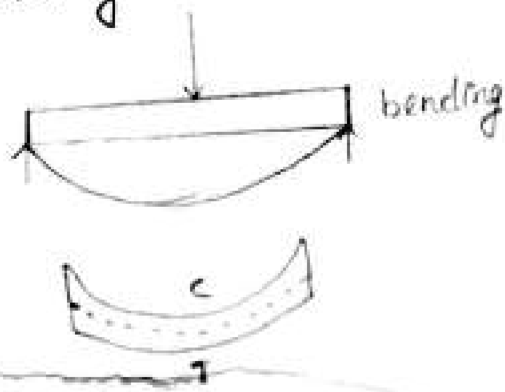
COLUMNS AND STRUTS

Beam



Longitudinal direction (length)

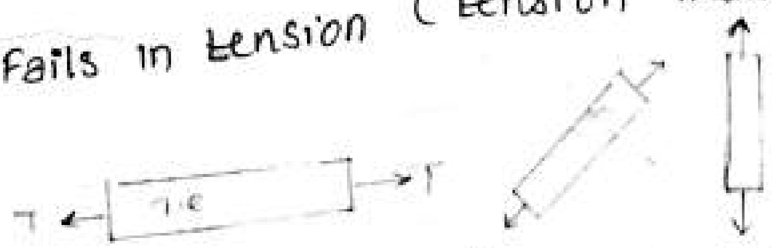
Bending



* Beams fails in transverse shear.

Tie member

* Fails in tension (tension member)



* Horizontal, Inclined or vertical

eg :- Roof truss, bridge truss, scaffold.

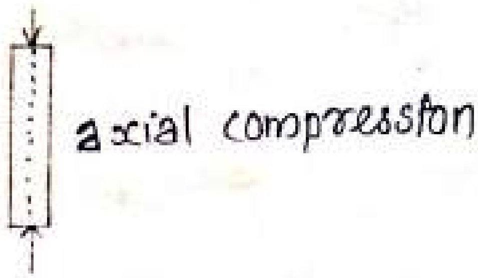
Struts

* Axial compresive forces/loads (compression members)
* Inclined, horizontal, vertical.
* It fails in compression

Example :- Roof, scaffold, bridges.

column

* columns are special case of struct in which struct is vertical.



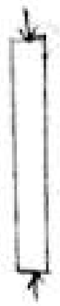
* fails in compression
* compression member.

columns are divided in to three

short columns

* fails in crushing.

intermediate columns



* It fails by compination coushing and Buckling

Long Columns

* Long columns are fails in buckling.

Failure of column & Struct

* when the axial compressive load

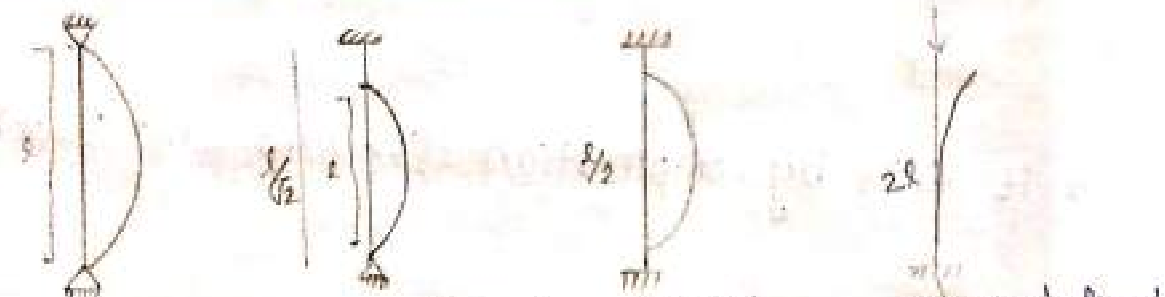
* The colounn will reach a point in which it reaches its maximum (ultimate) coushing structs. when the load is again increasing

* coushing load
The load ~~at~~ which the column ~~sto~~ crushes
in case of long columns failure occures due to buckling (sudden sideways ~~buckling load~~)

...ckling load

...e load at which the column buckling.

...pes of end conditions



...oth ends ...inged / pinned — one end hinged one end fixed — both end fixed — one end fixed other end free.

Euler's Formula (long columns)

Euler's buckling load $P_E = \frac{\pi^2 EI}{l_e^2}$

$E \rightarrow$ modules of elasticity

$I \rightarrow$ MI

$l_e \rightarrow$ Effective length.

End conditions	effective length (l_e)	Euler's buckling load, P_E
Both ends are hinged.	l	$P_E = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{l^2}$
	$\frac{l}{\sqrt{2}}$	$P_E = \frac{\pi^2 EI}{(l/\sqrt{2})^2} = \frac{\pi^2 EI}{l^2/2} = \frac{2\pi^2 EI}{l^2}$
	$\frac{l}{2}$	$P_E = \frac{\pi^2 EI}{(l/2)^2} = \frac{4\pi^2 EI}{l^2}$
	$2l$	$P_E = \frac{\pi^2 EI}{[illegible]} = \frac{\pi^2 EI}{[illegible]}$

A solid rectangular column of length 4 m is having a cross section of 200 x 100 mm. If the ends of column are hinged, find the Eulers crippling load. $E = 200\ kN/mm^2$

$l \rightarrow$ actual length

$l_e \rightarrow$ effective or equivalent length.

given that

$E = 200\ kN/mm^2$

$cs = 200 \times 100$

$l = 4m$

$$I = \frac{bd^3}{12} = \frac{200 \times 100^3}{12}$$

$$=$$

$$P_E = \frac{\pi^2 EI}{l_e^2}$$

$$= \frac{\pi^2 \times 200 \times 10^3 \times \left(\frac{200 \times 100^3}{12}\right)}{4000^2}$$

$$= \underline{\underline{2056,167.58}}$$

A column 12 m long has a section $1\ m^2$. the column is made of a metal having $E = 200 \times 10^3$ use Euler's formula to determine the buckling load.

* Both ends of the coloumns are pined
* one end is fixed other end is force.

i $P_E = \frac{\pi^2 E I}{l_e^2}$

$$= \frac{\pi^2 \times 200 \times 10^3 \, (8.333 \times 10)}{12000^2}$$

$$= \frac{1.644868}{144000.000}$$

$$= \underline{\underline{1142269632 \, N}}$$

ii $P_E = \frac{\pi^2 E I}{l_e^2}$

$$l = l/\sqrt{2}$$

$$= \frac{12000}{\sqrt{2}}$$

$$I = \frac{bd^3}{12} = \frac{1000 \times 1000^2}{12} = 8.333 \times 10^{10} \, mm^4$$

$$E = 200 \, kN/mm^2$$
$$= 200000 \, N/mm^2$$

$$P_E = \frac{\pi^2 \times 200000 \, N/mm^2 \times 8.333 \times 10^{10} \, mm^4}{\frac{12000^2}{\sqrt{2}}}$$

$$P_E = 2284539263 \, N$$

- A hollow alloy ~~tu~~ tub 4m has external an internal diameter 40 mm an 25 mm respectively. Find the Euler's crippling load for the ~~t~~ cube when both end are. $E = 70 \times 10^3 \, N/mm^2$.

$$I = \frac{\pi (D^4 - d^4)}{64}$$

$$P_E = \frac{\pi^2 E I}{l_e^2}$$

$$I = \frac{\pi (D^4 - d^4)}{64}$$

$$= \frac{\pi (40^4 - 25^4)}{64}$$

$$= \frac{3.14 (256 - 16)}{64}$$

$$= \cancel{39896.48}$$

$$= \underline{106488.94}$$

$$P_E = \frac{\pi^2 \times 70 \times 10^3 \times 106488.94}{4000^2}$$

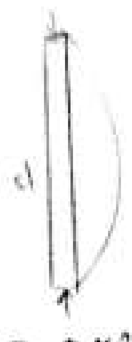
$$= \underline{\underline{4598.141507}}$$

=

Slenders ratio

$$\text{slenderness ratio} = \frac{\text{actual length}}{\text{least radius of gyration}}$$

$$= \frac{l}{k} = \frac{l}{\sqrt{I/A}}$$



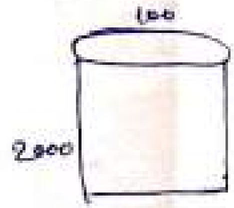
$I = AK^2$

$k = \sqrt{\frac{I}{A}}$

when buckling

$\frac{l}{k} \leq 80$ to avoid buckling.

* calculate the S r the column of $l_e = 2m$ and $D = 100\ mm$



$$I = AK^2$$

$$K = \sqrt{I/A}$$

$$I = \frac{\pi d^4}{64}$$

$$A = \frac{\pi d^2}{4}$$

$$k = \sqrt{\frac{\frac{\pi d^4}{64}}{\pi d^2/4}}$$

$$= \sqrt{d^2/16}$$

$$= \sqrt{\frac{100^2}{16}}$$

$$= \frac{100}{4}$$

$$= \underline{\underline{25}}$$

$$S.r \quad l/k$$

$$= \frac{2000}{25}$$

$$= \underline{\underline{80}}$$

Limits of Euler's Formula

* Euler's formula is not applicable for short column good for long columns only.
* No distingtion between crushing and buckling.

Rankine's Formula

- To overcome the drawback's of Euler's formula. ~~Rankine's~~ put forward Rankine's formula.

* It is applicable to both short and long column
* It considers crushing and buckling stresess.

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$$

Rankine's load = crushing load + Euler's buckling load.

Short columns

$l_e \downarrow \quad P_E \uparrow \quad \frac{1}{P_E} \downarrow$

$\frac{1}{P_E}$ is negleted becose $1/P_E \simeq 0$

$$\therefore \frac{1}{P} = \frac{1}{P_c} + 0$$

$$P = P_c$$

long columns

$l_e \uparrow \quad P_E \downarrow \quad \frac{1}{P_E} \uparrow$

so neglected P_c (which is comparitively very small)

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$$

$$\frac{1}{P} = 0 + \frac{1}{P_E}$$

$$P = P_E$$

Rankine's formula using Euler's load

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$$

$$\frac{1}{P} = \frac{P_E + P_c}{P_c \cdot P_E}$$

$$P = \frac{P_c \cdot P_E}{P_E + P_c}$$

$$= \frac{P_c \cdot P_E}{P_E\left(1 + \frac{P_c}{P_E}\right)}$$

$$= \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c \cdot A \cdot l_e^2}{\pi^2 E I}}$$

$$= \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c}{\pi^2 E} \cdot \frac{A l_e^2}{A k^2}}$$

$$P = \frac{\sigma_c A}{1 + \alpha\left(\frac{l_e}{k}\right)^2}$$

$\sigma_c = F/A$

$F = \sigma_c \cdot A$

$I = Ak^2$

P_c = crushing load

$= \sigma_c \cdot A$

$P_E = \frac{\pi^2 E I}{l^2}$

α – Rankine's constant

l_e – equivalent length

k – least rad of gyr

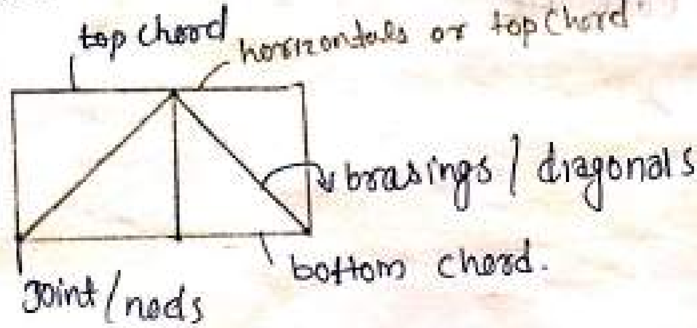
Material	σ_c N/mm²	Rankine's constant
wrought Iron	250	1/9000
cast Iron	550	$\frac{1}{6000}$
mild steel	320	$\frac{1}{7500}$
Timber	50	$\frac{1}{750}$

- A steel strut is of 150 mm. external diameter 10 mm thick. It is 3m long with pin jointed ends. find the load they can be equalent length

It is the ~~be~~ length of a ~~equalent~~ equivalent colom of the same material and cross section with hinged ends and having buckling load equal to that of given colomn.

TRUSS

A truss is a structure comprising one or more triangular units, constructed with straight members whose ends are connected at joints or ~~rots~~ nods.



Types of trusses

Types of trusses classified in to three.

* Efficeent / perfect

~~when a truss is load~~

For a

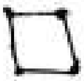
$m = 2j - 3$

$m \rightarrow$ members

$j \rightarrow$ Joints / nodes

Eg Δ $m = 2j - 3$

$3 = 2\times3 - 3 = \underline{3}$



$m = 4$
$J = 4$

$4 = 2 \times 4 - 3$
$\neq 5$

* Deficient | imperfect

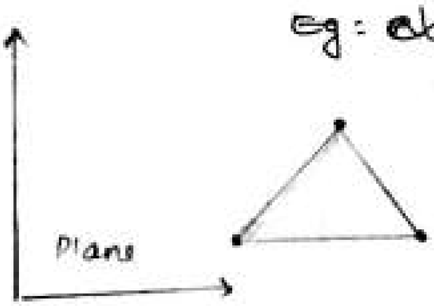
$m < 2j - 3$

* Redundant

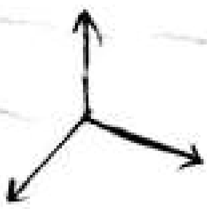
$m > 2j - 3$

→ members and joints same plane is called plane ~~tr~~ trusses. 2D

eg: ~~electrical~~, bridges, roof, wave houses railway



→ In the case of tower it is different plane so we - called as space trusses. 3D in space.



eg: ~~bridges, roof~~, wave ~~houses~~, ~~Rai~~
: electrical transmision towers.

Analysis of truss

Before analysing a member

assumptions

* All loads acts at joints only

* only axial force are acting

* Self weight is neglected

* The truss is a perfect truss ies it is obey

* All members ~~are~~ have same cross section

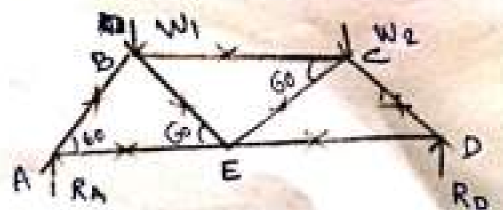
* All the members are straight

methods of Analaysis

Analysis in case of trusses means to determine the avoid forces

1. Graphical method
2. Analytical method
   * method of sections
   * method of Joints.

method of Joints



① Reactions

$R_A + R_D = W_1 + W_2$

$\Sigma M = 0$

Equations of statics

$\Sigma H = 0 \quad \Sigma f_x = 0$

$\Sigma V = 0 \quad \Sigma f_y = 0$

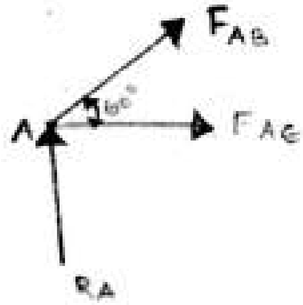
Now we are select any joint but one condition two unknown joints.

② Mark T/C

③ Select a joint

* only two unknowns.

Joint A



* FBD of Joint A
* Assume member force (C/T)

Compression Tension

* Mark in FBD (free body diagram)

④* Resolve force & solve

$\Sigma F_y = 0$

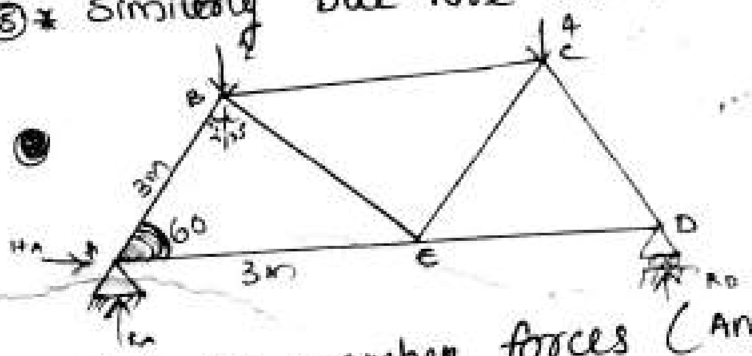
$R_A + F_{AB} \sin\theta = 0$

$R_A \rightarrow$ known quantity
$\theta \rightarrow$ " "

$\Sigma F_x = 0.$

$F_{AE} + F_{AB} \cos\theta = 0$

⑤* Similarly Due fore all Joints. ...



Find the member forces (Analys the given truss?)

→ Find support reactions

$R_A + R_D$

$\Sigma F_y = 0$

$R_A + R_D = 2 + 4 = 6$

$\Sigma F_x = 0$

$H_A = 0$

$$\Sigma M_A = 0$$

$$2 \times 1.5 + 4 \times 4.5 = R_D \times 6$$

$$R_D 6 = 21$$

$$R_D = 21/6$$

$$= \underline{\underline{3.5}} \text{ kN}$$

$$R_A = 2.5 \text{ kN}$$

(3) Mark FBD of joint A member forces.

JOINT A

$$\Sigma F_y = 0$$

$$R_A + F_{AB} \sin\theta$$

$$2.5 + F_{AB} \sin 60 = 0$$

$$F_{AB} = \frac{-2.5}{\sin 60}$$

$$= \underline{\underline{-2.89 \text{ kN}}}$$

$$\Sigma F_x = 0$$

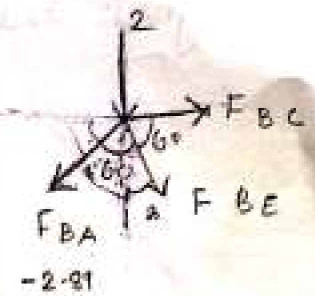
$$F_{AE} + F_{AB} \cos\theta$$

$$F_{AE} = -F_{AB} \cos 60$$

$$= -(-2.89) \times \cos 60$$

$$= \underline{\underline{1.44 \text{ kN}}}$$

JOINT - B



$$\Sigma F_y = 0$$

$$2 + F_{BA} \cos 30 + F_{BE} \cos 30 = 0$$

$$2 + (-2.89) \cos 30 + F_{BE} \cos 30 = 0$$

$$F_{BE} = \frac{2 + 2.89 \cos 30}{\cos 30}$$

$$= \underline{+0.580}$$

$\Sigma F_x = 0$

$$F_{BC} + F_{BE}\cos 60 = F_{BA}\sin 30$$

$$F_{BC} + 0.58 \times \cos 60 = -2.89 \sin 30$$

$$F_{BC} = \underline{\underline{-1.735\ kN}}$$

$F_{AC} = 1.44$

$F_{AB} = -2.89$

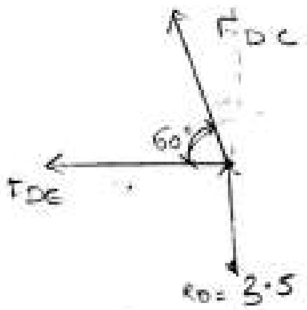
$F_{BE} = 0.58$ kN

$F_{BC} = 1.735$ kN

$F_{DC} = -4.041$

$F_{DE} = 2.02$

JOINT - D



$\Sigma F_y = 0$

$$3.5 + F_{DC}\sin 60 = 0$$

$$F_{DC} = \frac{-3.5}{\sin 60}$$

$$= \underline{\underline{-4.041\ kN}}$$

$\Sigma F_x = 0$

$$F_{DE} + F_{DC}\cos 60 = 0$$

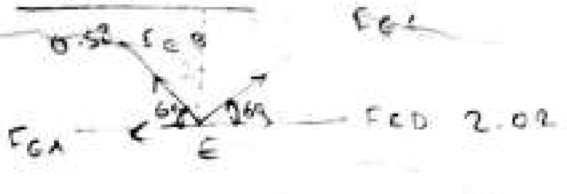
$$F_{DE} = -F_{DC}\cos 60$$

$$= -4.041 \times \cos 60$$

$$= \underline{\underline{2.02\ kN}}$$

Member	Force (kN)	Nature
AB	2.89	C
AE	1.45	T
BE	0.58	T
BC	1.735	C
EC	0.56	C
DC	4.041	C
ED	2.02	T

JOINT E



$$F_{EB}\cos 30 + F_{EC}\cos 30$$

$$0.58 \cdot \cos 30 = F_{EC}\cos 30$$

$$F_{EC} = 0.56$$

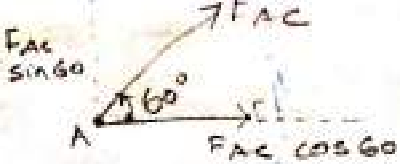
$\Sigma F_x = 0$

$$F_{CA} + F_{ED} + F_{EB}\cos 60 + F_{EC}\cos 60 = 0$$

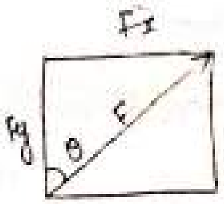
$$1.44 + F_{ED} + 0.58 \times \tfrac{1}{2} + 0.56 \times \tfrac{1}{2} = 0 \qquad F_{ED} = 2.02$$

Scanned by CamScanner

①



②

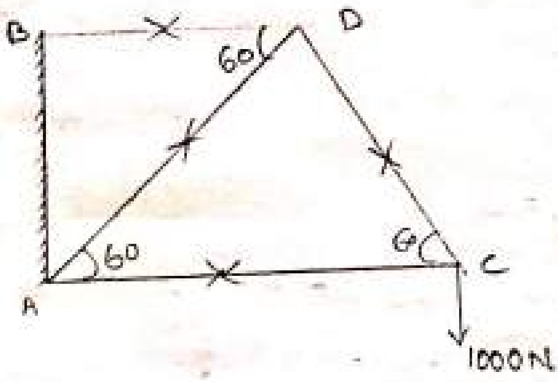


$$\sin\theta = \frac{f_x}{F}$$

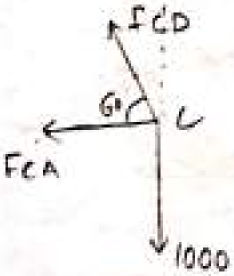
$$\boxed{f_x = F \sin\theta}$$

$$F \sin 60 = F \cos 30$$

②



Joint C



$$\Sigma f_y = 0$$

$$1000 = F_{CD} \sin 60$$

$$F_{CD} = \sin 60$$

$$\frac{1000}{\sin 60} = F_{CD}$$

$$= \underline{\underline{1150.7 \text{ N}}}$$

$$\Sigma f_x = 0$$

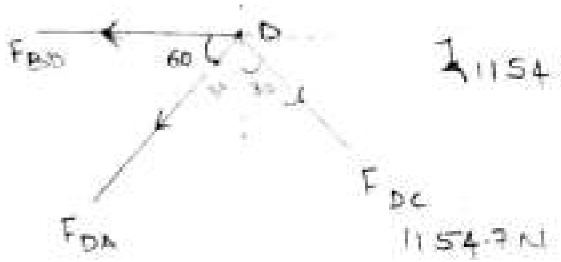
$$= f_{CD} \cos 60 + F_{CA}$$

$$= 1150.7 \cos 60 + F_{CA}$$

$$= \underline{\underline{-577.35}}$$

Scanned by CamScanner

JOINT : D



$\Sigma f_y = 0$

$1154.7 \cos 30 = F_{DA} \cos 30$

one method

$\underline{1154.7 = F_{DA}}$

another method

$\Sigma F x = 0$

$0 = F_{DA} \cos 30 + F_{DC} \cos 30$

$F_{BD} + F_{DA} \cos 60 = F_{DC} \sin 30$

$= F_{DA} \cos 30 + 1150.7 \cos 30$

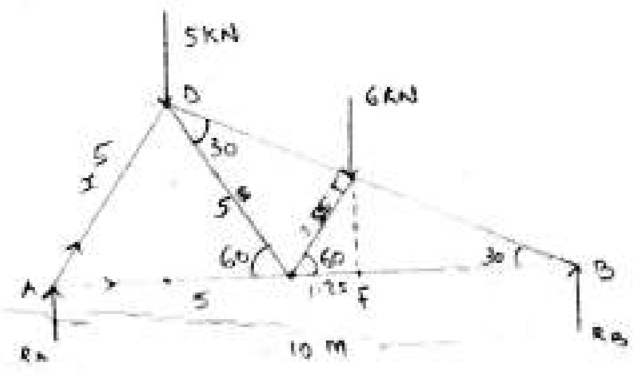
$F_{BD} + -1150.7 \times \cos 60 = 1150.7 \times \sin 30$

$F_{DA} = 1154.7\ N$

$F_{BD} = \underline{1154.7\ N}$

Member	force(N)	Nature
AD	1150.7	C
BD	1154.7	T
DC	1150.7	T
AC	577.35	C

③



$\sin 30 = \frac{x}{5}$

$x = \sin 30 \times 5$

$= \underline{2.55}$

$\sin 30 = x/10$

$x = 10 \times 1/2 = \underline{5}$

$\cos 60 = \frac{x}{2.5}$

$x = 2.5 \times 1/2$

$= \underline{1.25}$

$\Sigma f_y = 0$

$R_A + R_B = W_1 + W_2$

$5 + 6 = 11$

$\Sigma F x = 0$

$H_A = 0$

$\Sigma M_A = 5 \times 2.5 + 6 \times 6.25 = R_B\ 10$

$50 = R_B 10$

$R_B = \frac{50}{10} = \underline{5}$

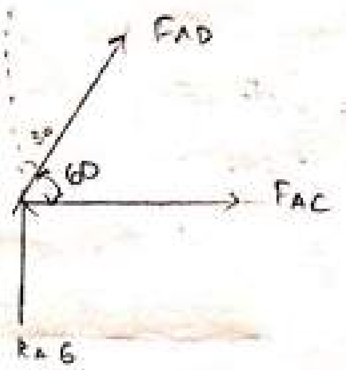
$R_A + R_B = 11$

$R_A + 5 = 11$

$R_A = 11 - 5$

$= \underline{\underline{6}}$

JOINT - A



$\Sigma F_y = 0$

$6 + F_{AD} \cdot \cos 30 = 0$

$F_{AD} = \frac{-6}{\cos 30} = \underline{\underline{-6.928}}$

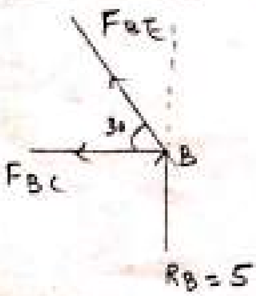
$\Sigma F_x = 0$

$F_{AC} + F_{AD} \cos 30 = 0$

$F_{AC} = +6.928 \cos 60$

$= \underline{\underline{3.46}}$

JOINT B



$\Sigma F_y = 0$

$5 + F_{BE} \sin 30 = 0$

$F_{BE} = \frac{-5}{\sin 30} = \underline{\underline{-10}}$

$\Sigma F_x = 0$

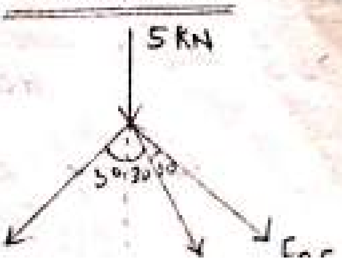
$F_{BC} + F_{BE} \cos 30 = 0$

$- F_{BC} = -10 \cos 30$

$+ F_{BC} = -8.66$ KN

$\underline{\underline{F_{BC} = 8.66 \text{ KN}}}$

JOINT D



$\Sigma F_y = 0$

$5 + F_{DE} \cos 60$

$$5 + F_{DA}\cos 30 + F_{DC}\cos 30 + F_{DE}\cos 60 = 0$$

$$5 + -6.928\cos 30 + F_{DC}\cos 30 + F_{DE}\cos 60 = 0$$

$$-6.928 \times \sin 30 = F_{DC}\sin 30 + F_{DE}\sin 60$$

$$-3.4625 = 0.5\,F_{DC} + 0.866\,F_{DE} \quad \text{— ①}$$

$$0.866\,F_{DC} + 0.5\,F_{DE} = 1 \quad \text{— ②}$$

$$-3 = 0.433\,F_{DC} + 0.75\,F_{DE} \quad \text{— ③}$$

$$0.433\,F_{DC} + 0.25\,F_{DE} = 0.5 \quad \text{— ④}$$

③ − ④ ⇒

$$0.433\,F_{DC} + 0.75\,F_{DE} = -3$$

$$0.433\,F_{DC} + 0.25\,F_{DE} = 0.5$$

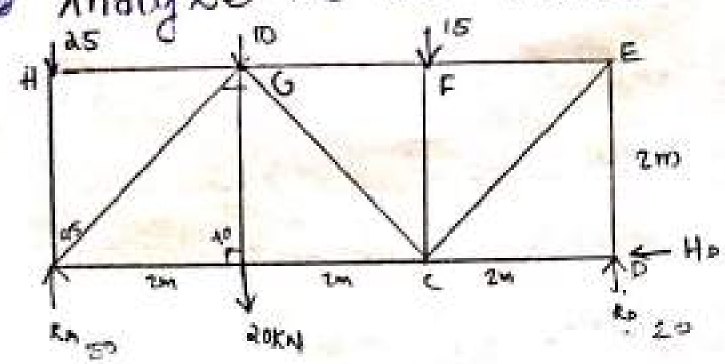
$$0.5\,F_{DE} = -3.5$$

$$F_{DE} = \frac{-3.5}{0.5} = \underline{\underline{-7}}$$

METHODS OF SECTIONS

* It is usefull when forces in a few members of a frame are required to found.
* Number of unknown forces doesn't exeed three
* A section line ~~is~~ drawn through required section
* Apply equilibrium eqns ($\Sigma F_y = 0$ $\Sigma F_x = 0$ $\Sigma M = 0$) and solve

Analyze the ~~truss~~ frame



$$R_A + R_D = W_1 + W_2$$

$$= 20 + 25 + 10 + 15 = 70$$

$$\Sigma M_A = 0$$

$$\cancel{R_A \times 0 + 25 \times 0 +}$$

$$R_A \times 0 + R_D \times 6 = 25 \times 0 + 10 \times 2m + 20 \times 2 + 15 \times 4$$

$$R_D 6 = 120$$

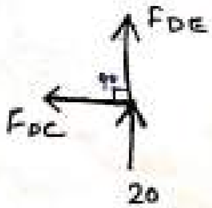
$$R_D = \frac{120}{6}$$

$$= \underline{\underline{20}}$$

$$R_A + 20 = 70$$

$$R_A = 70 - 20 = \underline{\underline{50}}$$

JOINT D

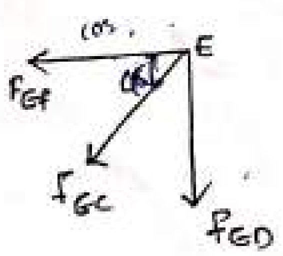


$$20 + F_{DE} = 0$$

$$F_{DE} = \underline{\underline{-20}}$$

$$F_{DC} = \underline{\underline{0}}$$

JOINT E



$$F_{ED} + F_{EC} \sin 45 = 0$$

$$-20 + F_{EC} \sin 45 = 0$$

$$F_{EC} = \frac{20}{\sin 45}$$

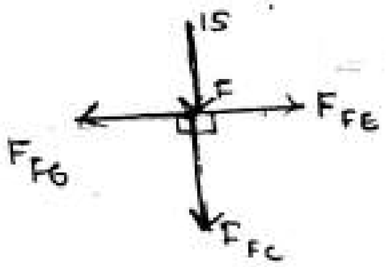
$$= \underline{\underline{28.28}}$$

$F_{EF} + F_{EC} \cos 45 = 0$

$F_{EF} + 28.28 \cos 45 = 0$

$F_{EF} = \cancel{28} - \underline{\underline{19.99}} = \underline{20} \text{ kN}$

JOINT F



$15 + F_{FC} = 0$

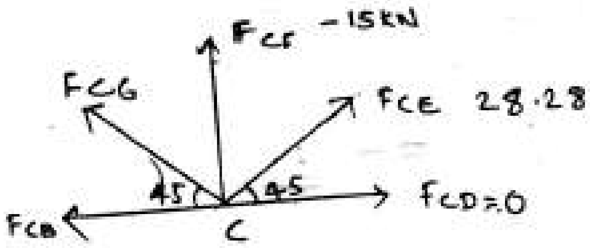
$F_{FC} = \underline{\underline{-15}} \text{ kN}$

$F_{FG} = F_{FE}$

$F_{FG} = \underline{\underline{-20 \text{ kN}}}$

JOINT C

$\Sigma F_y = \cancel{0 + 28.2}$



$F_{CE} \sin 45 + F_{CF} + F_{CG} \sin 45 = 0$

$28.28 \sin 45 + -15\text{kN} + F_{CG} \sin 45 = 0$

$28.28 \sin 45 + -15 \text{ kN} + F_{CG} \sin 45 = 0$

$19.99 + -15\text{kN} + F_{CG} \sin 45 = 0$

$F_{CG} = \dfrac{-19.99 + 15}{\sin 45} = -7.056$

$\Sigma F_x = 0$

$F_{CD} + F_{CE} \cos 45 = F_{CB} + F_{CG} \cos 45$

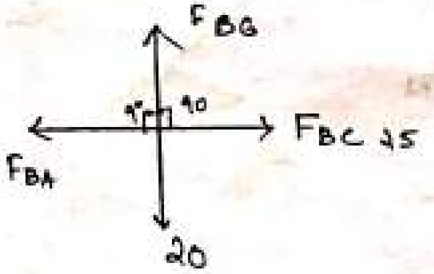
$0 + 28.28 \cos 45 = F_{CB} + -7.056 \cos 45$

$20 = F_{CB} + -7.056 \cos 45$

$F_{CB} = -7.056 \cos 45 - 20$

$= \underline{\underline{25 \text{ kN}}}$

JOINT B



$\Sigma f_y = 0$

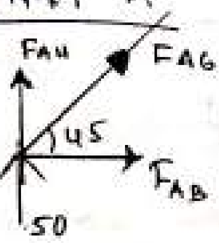
$F_{BG} = 20$

$\Sigma f_x = 0$

$F_{BC} = F_{BA}$

$25\ KN = F_{BA}$

JOINT A



$\Sigma F_y = 0$

$F_{AB} + F_{AG} \sin 45 + 50 = 0$

$F_{AH} = -50 + 35.35 \sin 45$

$= -25$

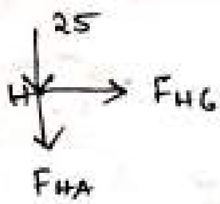
$\Sigma F_x = 0$

$F_{AB} + F_{AG} \cos 45 = 0$

$25 + F_{AG} \cos 45 = 0$

$F_{AG} = -\underline{35.35}$

JOINT H



$25 + F_{HA} = 0$

$F_{HA} = -25$

$F_{HG} = 0$

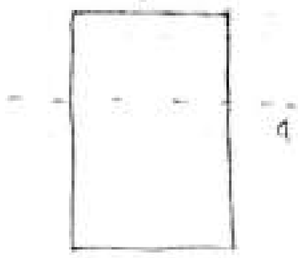
Member	Force	Nature
AB		
AH		
HG		
AG		
GB		
BC		

MODULE II

section Modulus

section modulus $z = \frac{I}{y} = \frac{\frac{bd^3}{12}}{d/2} = \frac{bd^2}{6}$

Rectangular section

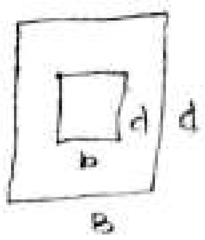


$$z = \frac{I}{y} = \frac{\frac{bd^3}{12}}{d/2} = \frac{bd^2}{6}$$

$$I = \frac{bd^3}{12}$$

$$\cancel{y} =$$

Hollow rectangular section



$$SM\ z = \frac{I}{y}$$

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{BD^3 - bd^3}{12}$$

$$y = D/2$$

$$z = \frac{I}{y} = \frac{\frac{BD^3 - bd^3}{12}}{D/2} = \frac{BD^3 - bd^3}{6D}$$

Circular Section

$\frac{\pi d^4}{64}$ $z = \frac{\pi d^3}{32}$ $y = d/2$ $\frac{\pi(D^4 - d^4)}{32D}$

$z = I/y$

$I = \frac{\pi(D^4 - d^4)}{64}$ $\therefore y = D/2$

Scanned by CamScanner

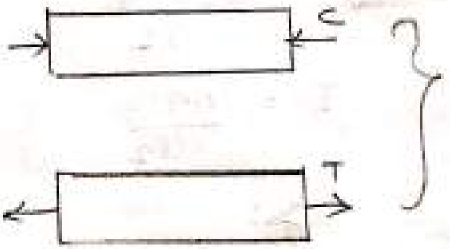
* Hollow circular section

$$I = \frac{\pi D^4}{64} - \frac{\pi d^4}{64}$$

$$Z = \frac{\frac{\pi D^4}{64} - \frac{\pi d^4}{64}}{D/2}$$

$$Z = \frac{\pi (D^4 - d^4)}{32D}$$

DIRECT AND BENDING STRESSES



direct C/T stresses

bending stress

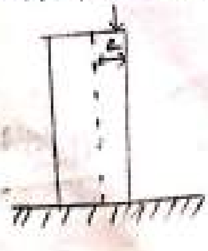
Axial load

When the load is acting along the longitudinal - axis of the column.

longitudinal axis of column

$$\sigma \text{ or } f = \frac{\text{force}}{\text{Area}} = \frac{P}{A}$$

eccentric load

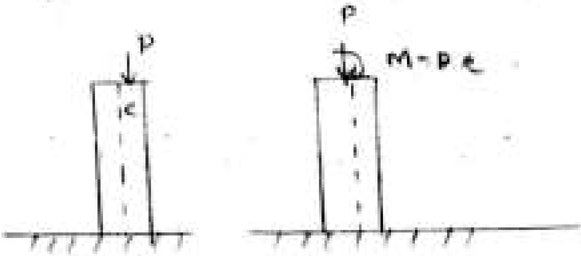


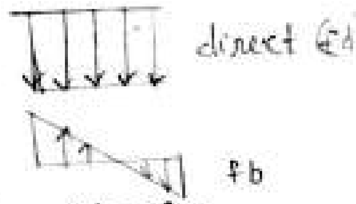
$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

does not coincide with the axis of a column.

eccentricity

The horizontal distance b/w the longitudinal axis of coloumn and line of action load.





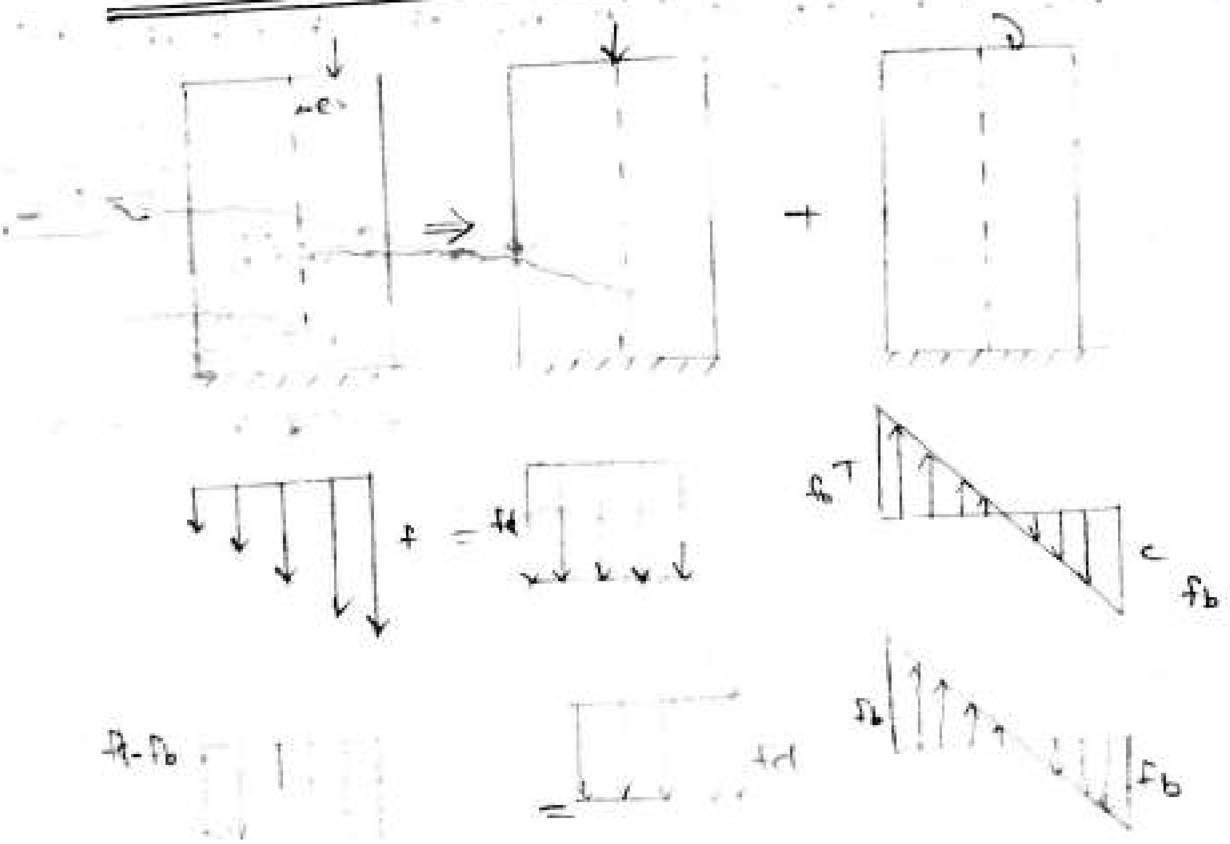
combined stress = direct stress + bending stress

$$= \frac{P}{A} \pm \frac{M}{Z}$$

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

$$f = \frac{My}{I} = \frac{M}{I/y} = \frac{M}{Z}$$

Stresses in column due to Eccentric loading



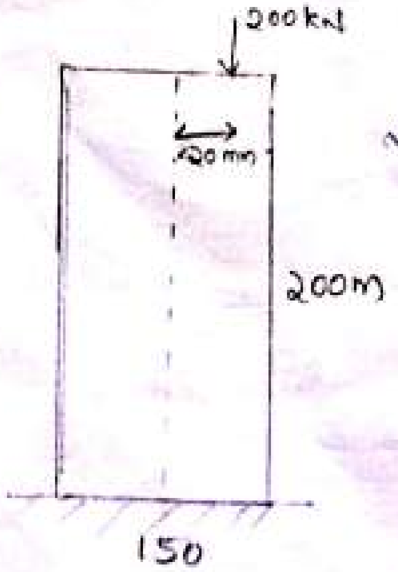
Minimum and maximum stresses

When a column is subjected to eccentric the edge of the column towards the eccentricity will be subjected to maximum stress. (F_{max}) and the opposite edge will be subjected to minimum stress.

$$F_{max} = f_d + f_b$$
$$= P/A + \frac{M}{Z}$$

$$f_{min} = F_d - f_b$$
$$= P/A - \frac{M}{Z}$$

Q) A rectangular strut is 200mm wide and 150 mm thickness. It carries a load of 200 kN. At an eccentricity of 20 mm in a plane bysecting the thickness. Find the maximum and minimum intensity of stress in the section distribution diagram.



$\boxed{M = P.e}$

$$f_{max} = fd + fb$$
$$= P/A + \frac{M}{Z}$$
$$= \frac{200\times10^3}{150\times200} + \frac{(200\times10^3)\times2}{}$$

$$F_{max} = \frac{200 \times 10^3}{150 \times 200} + \frac{4 \times 10^6}{1 \times 10^6}$$

$$= \frac{200 \times 10^3}{30000} + \frac{4 \times 10^6}{1 \times 10^6}$$

$$= 6.666 + 4$$

$$= \underline{\underline{10.67 \ N/mm^2}}$$

$M = P.e$

$= (200 \times 10^3) \times 20$

$= 4000000$

$= \underline{\underline{4 \times 10^6 \ Nmm}}$

$Z = \frac{I}{y}$

$= \frac{bd^3/12}{d/2}$

$= bd^2/6$

$= \frac{150 \times 200^2}{6}$

$= \underline{\underline{1 \times 10^6}}$

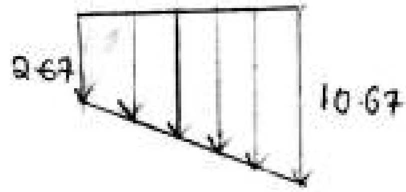
$$F_{minimum} = F_d - F_b$$

$$= P/A - \frac{M}{Z}$$

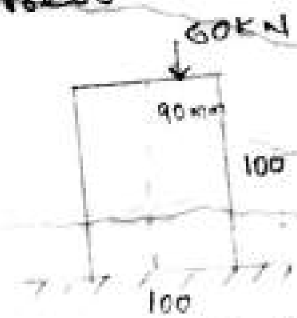
$$= \frac{200 \times 10^3}{150 \times 200} \ a - \frac{4 \times 10^6}{1 \times 10^6}$$

$$= 6.666 - 4$$

$$= \underline{\underline{2.67}}$$



- A short column 100 x 100 mm ~~to~~ is subjected to an eccentric load of 60 kN at an eccentricity of 40mm in the plane bisecting the two opposite faces. Find the maximum & minimum intencity of - faces. find the maximum & minimum intencity of stress at the BASE section.?



$M = P.e$

$= (60 \times 10^3) 40$

$= 24 \times 10^5$

$Z = \frac{I}{y}$

$= bd^2/6$

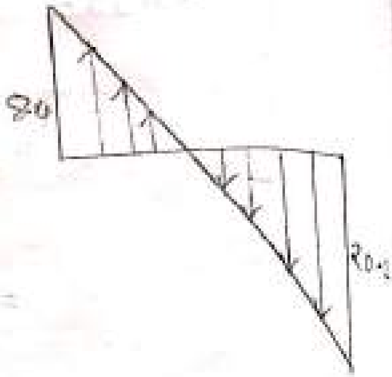
$= \frac{100 \times 100^2}{6}$

$= 16666$

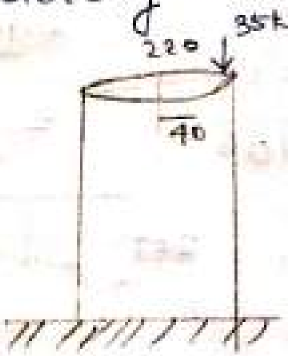
$$F_{max} = \frac{60 \times 10^3}{100 \times 100} + \frac{24 \times 10^5}{166666.6}$$

$$= \underline{\underline{20.4 \ N/mm^2}}$$

$F_{minm} = -8.4\ N/mm^2$



5. A circular column 220 mm diameter carries a vertical load of 35 kN at a distance of 40 mm from the axis. find the max and minimum stress intencity at the base ?



$$Z = \frac{I}{y} = \frac{\frac{\pi d^4}{64}}{d/2} = \frac{\pi d^3}{12}$$

$$F_{max} = P/A + \frac{M}{Z}$$

$$Z = \frac{\pi d^3}{12} = \frac{\pi \times 40^3}{12} =$$

$$M = P.e =$$

$$z = \frac{\pi d^2}{32}$$

$$F_{max} = P/A + M/z$$

$$= \frac{35 \times 10^3}{\pi \times 10^3} + \frac{35 \times 40 \times 10^3}{\pi \times \frac{20^2}{32}}$$

$$= \underline{\underline{2.62\ N/mm^2}}$$

$$F_{minm} = P/A - \frac{M}{Z}$$

$$= \frac{35 \times 10^3}{\pi \times 110} - \frac{35 \times 10^3 \times 40}{\pi \times 220^3/32}$$

$$= \underline{\underline{-0.42\ N/mm^2}}$$

Scanned by CamScanner

A short column of external and internal diameter 200 and 150 mm respectively carries an eccentric load for 40 kN at a distance of 85 mm. Find maximum and minimum stress ?

$F_{max} = \frac{P}{A} + \frac{M}{Z}$

$= \frac{40\times10^3}{31415.92} + \frac{34\times10^5}{536893.0}$ $M = P.e$

$= 2.91026 + 6.3327$ $= (40\times10^3)\,85$

$= 34\times10^5$

$= \underline{9.2429}$

$Z = \frac{I}{y} = \frac{\frac{\pi D^4 - \pi d^4}{64}}{D/2}$

$= \frac{\pi(D^4 - d^4)}{32D}$

$= \frac{\pi\times(200^4 - 150^4)}{32\times200}$

$= 536893.2$

$A = \frac{\pi D^2}{4} - \frac{\pi d^2}{4}$

$= \frac{\pi\times200^2}{4} - \frac{\pi\times150^2}{4}$

$= 13744.46$

$F_{minim} = \frac{P}{A} - \frac{M}{Z}$

$= 2.91026 - 6.3327$

$= \underline{-3.42244}$

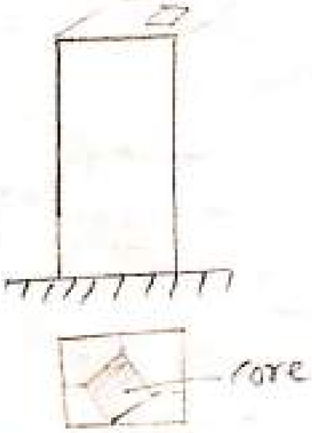
Limit of eccentricity

The minimum distance of load from C.G such that if load act with in this distance there is no tension in the column. The maximum distance is called limit of eccentricity or critical eccentricity.

IMP

CORE OF A SECTION

When an eccentric load acts in such a way the on region around C-G of the Section - No- tensile stress is develop any were in the region then the area of that region is called core of the section or kernel

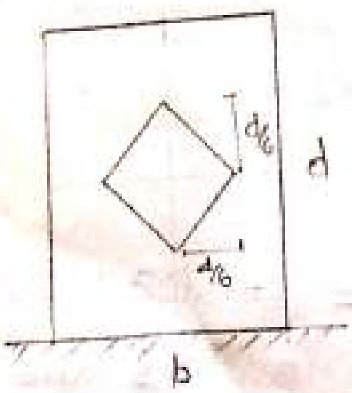


No tension condition
Bending stress ≤ direct stress.

$$f_b \leq f_d$$

$$\frac{M}{Z} \leq P/A$$

Limit of eccentricity for rectangular section.



→ for no tension condition $f_b \leq f_d$

$$\frac{M}{Z} \leq P/A$$

$$\frac{P.e}{\ } \leq \frac{P}{A}$$

$$\frac{P\cdot e}{\frac{bd^2}{6}} \le \frac{P}{b\cdot d}$$

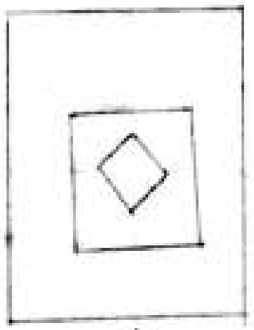
$$\frac{e}{\frac{\cancel{b}d^{2}}{6}} \le \cancel{b}\cdot\cancel{d}$$

$$\frac{e}{d/6} \le 0$$

$$e \le d/6$$

~~Limi~~

Limit of eccentricity for hollow rectangular column



For no tension condition $f_b \le f_d$

$$f_b \le f_d$$

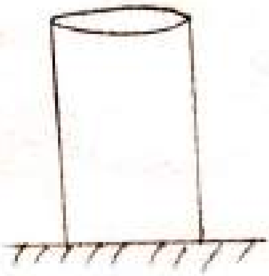
$$\frac{M}{Z} \le P/A$$

$$\frac{\cancel{P}\cdot e}{\left(\frac{BD^3 - bd^3}{6D}\right)} \le \frac{\cancel{P}}{BD - bd}$$

$$\frac{6De}{BD^3 - bd^3} \le \frac{1}{BD - bd}$$

$$e \le \frac{BD^3 - bd^3}{6D\,(BD - bd)}$$

Limit of eccentricity for circular column



$$f_b \le f_d$$

$$\frac{M}{Z} \le \frac{P}{A}$$

$$\frac{\cancel{P} e}{\frac{\pi d^3}{32}} \le \frac{\cancel{P}}{\cancel{\pi} r^2}$$

$$\frac{e}{\frac{d^3}{32}} \le \frac{1}{r^2} \qquad r^2 = \frac{d^2}{4}$$

$$\frac{e}{d/8} \le 0 \qquad \frac{\overset{8}{\cancel{32}} e}{d^{\cancel{3}}} \le \frac{1}{d^{\cancel{2}}/4}$$

$$\frac{e}{d/8} \le 0$$

$$\underline{e \le d/8}$$

Limit of eccentricity for hollow circular column

$$F_b \le F_d$$

$$\frac{M}{Z} \le \frac{P}{A}$$

$$\frac{P \cdot e}{\frac{\pi (D^4 - d^4)}{32 D}} \le \frac{P}{\frac{\pi (D^2 - d^2)}{4}}$$

$$\frac{e}{\frac{\pi (D^4 - d^4)}{32 D}} \le \frac{1}{\pi/4 \, (D^2 - d^2)}$$

$$\frac{8 e D}{D^4 - d^4} \le \frac{1}{D^2 - d^2}$$

$$e \le \frac{D^4 - d^4}{8 D (D^2 - d^2)}$$

$e \le ($

$e \le$

- A short
200 and
load of
which
base

$e \le D^2$

$e \le$

DAMS

V.imp

DAM

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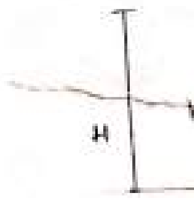


Forces

(i) weigh
centre

ii) pres

$$e \leq \frac{(D^2+d^2)(D^2-d^2)}{8D(D^2-d^2)}$$

$$e \leq \frac{D^2+d^2}{8D}$$

- A short column of external and internal diameter 200 and 150 mm respectively carries and eccentric load of 40 kN. Find the greatest eccentricity at which the load cannot develop any tension at the base

$$e \leq \frac{D^2+d^2}{8D}$$

$$e \leq \frac{200^2+150^2}{8\times200}$$

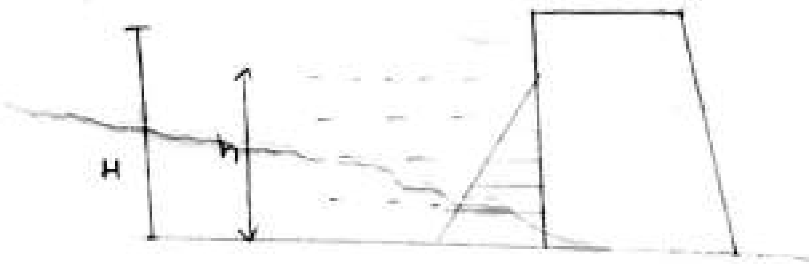
$$= \underline{\underline{39\,mm}}$$

DAMS AND RETAINING WALLS

V-IMP

DAM

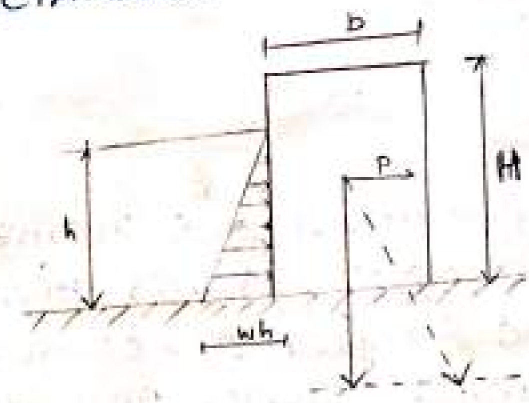
A structure build up of masonry or concrete to stope large quantities of water which is used for the purpose of Drinking Irrigation and power generation.



Forces Acting on the Dam

(1) weight of the Dam acting downwards through

RECTANGULAR SECTION



1. weight of dam / unit length

w - sp. weight of water

$W = \rho \times V$

$W = \rho \times A$

2. Total pressure of water

P = Area of assume diagram

$= \frac{1}{2} wh \cdot h$

$= \frac{wh^2}{2}$

3. $R = \sqrt{P^2 + W^2}$

4. Eccentricity

When weight of the dam acts downward and pressure of water acts horizontally. The resultant will be making an eccentricity (e) of the base of the dam equate moment about the points.

$W \times e = P \times h/3$

$e = P/W \times h/3$

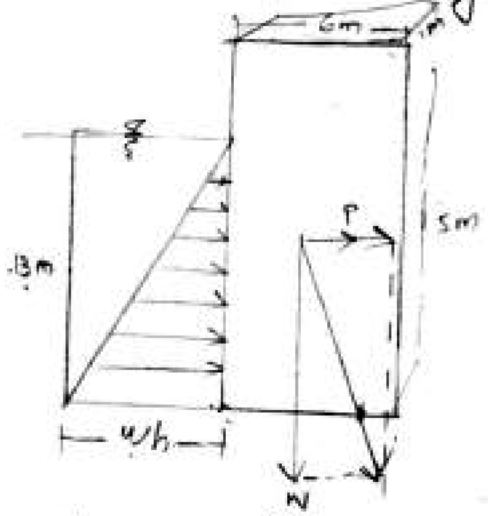
5. Fmax & Fmin

$F_{max} = F_d + F_b \qquad F_{min} = F_d - F_b$

$F_d = W/A = \frac{W}{b \times 1} = \frac{W}{b}$

$F_b = \frac{M}{Z} = \frac{W \cdot e \cdot y}{I} = \frac{W \cdot e \cdot b/2}{1 \times \frac{b^3}{12}} = \frac{6\,We}{b^2}$

- A concrete dam of rectangular cross section high - and 6m wide. contain water up to a a height of 13 m, Draw the p. daigram at the base, Assume - and 25 kN/m³ respectively



$W = V \times Sp.wt$
$= 15 \times 6 \times 1 \times 10$
$=$

ρ - specific wt of water

Given
$\rho = 25\ kN/m^3$
$w = 10\ kN/m^3$

① Weight of dam / unit length $= \rho \times A \times h = 25 \times 6 \times 15$
$= \underline{\underline{2250}}\ kN/m$

② Total pressure of water $p = \frac{1}{2} \times wh^2$
$= \frac{10 \times 13^2}{2} = \underline{\underline{345\ kN/m}}$

③ Resultant $R = \sqrt{p^2 + w^2} = \sqrt{845^2 + 2250^2} = 2403$

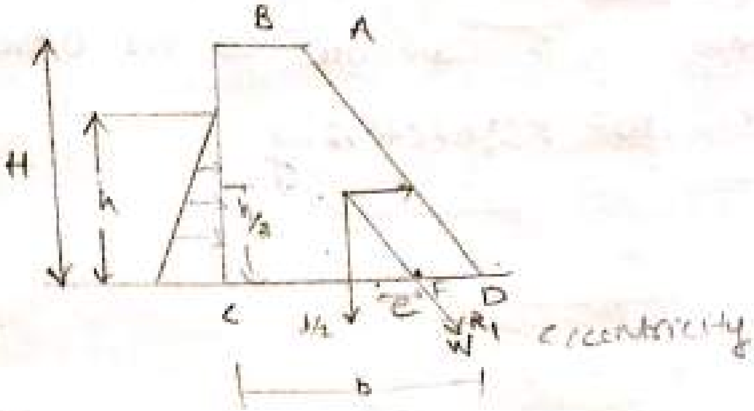
④ Eccentricity $(e) = P/w \times h/3 = \frac{845}{2250} \times 13/3 = 1.62$

⑤ $F_{max} = w/b\left(1 + \frac{6e}{b}\right) = \frac{2250}{6}\left(\frac{1 + 6 \times 0.62}{6}\right) = 982.5\ kN/m^2$

$F_{min} = w/b\left(1 - \frac{6e}{b}\right) = \frac{2250}{6}\left(\frac{1 - 6 \times 1.62}{6}\right) = -2385\ kN$

Resultand at a distance of (3 + 1.62) from A

TRAPIZOIDAL SECTION



1. weight of dam/unit length

$$W = \rho A$$

2. Total pressure of water

$$P = \frac{wh^2}{2}$$

3. Resultant thrust $R = \sqrt{W^2 x P^2}$

4. Eccentricity $e = CF - b/2$

$$= (CE + EF) - b/2$$

$$CE = \frac{a^2 + ab + b^2}{3(a+b)}$$

$$W \times EF = P \times h/3$$

$$EP = P/W \times h/3$$

5.

5 $F_{max} = W/b\left(1 + \frac{6e}{b}\right)$

$F_{min} = W/b\left(1 - \frac{6e}{b}\right)$

Q. A concrete dam of trapizoidal section having water on vertical phase 12m height. The base of the dam is 7mtr wide and top 3m wide

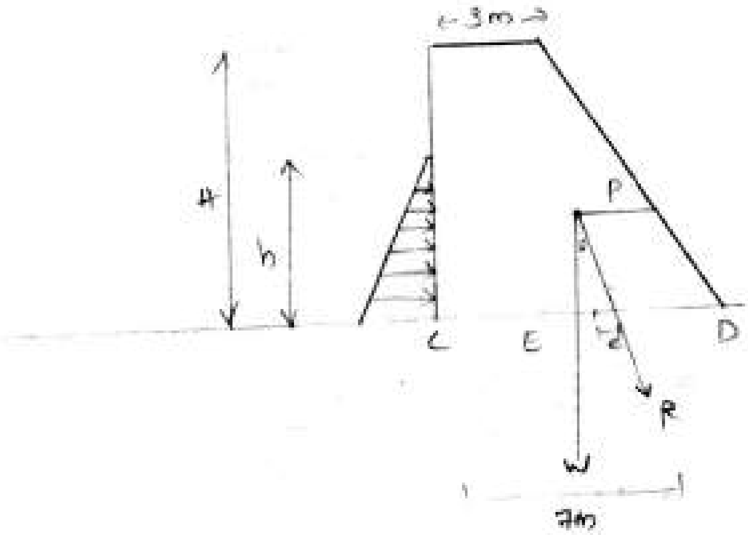
Find Resultand thrust on the base per meter length of the dam.

(b) The point were the resultant astruss cuts the base.

(c) Intencities of maximum and minimum stress across the base

Take weight of concrete has 25 kN/m³ and the water level coinside in with the top.

specific weight of water = 10 kN/m³



1. weight of dam / unit length

$$W = \rho A$$
$$= 25 \times \left[\frac{3+7}{2}\right] \times 12$$
$$= 1500$$

2. Total pressure of water $P = \frac{wh^2}{2}$

$$= \frac{10 \times 12^2}{2}$$
$$= 120 \text{ kN/m}$$

(3) Resultant ~~thrus~~ thrust, $R = \sqrt{W^2 + P^2}$

$$= \sqrt{1500^2 + 720^2}$$
$$= 1663.85 \text{ kN/m}$$

Eccentricity $e = CF - 7/2$

$= CE + EF - 3.5)$

$$CE = \frac{a^2 + ab^2 + b^2}{3(a+b)} = \frac{3^2 + 3\times 7 + 7^2}{3(3+7)}$$

$$= \underline{\underline{2.683}}$$

$$EF = P/W \times h/3 = \frac{720}{1500} \times 12/3 = 1.92$$

$$\therefore e = 2.683 + 1.92 - 3.5$$

$$= \underline{\underline{1.053\,m}}$$

∴ Resultant is cutting at a distance of [1.92+2.63

$$F_{max} = W/b \left[1 + \frac{6e}{b}\right]$$

$$= \frac{1500}{7}\left[1 + \frac{6\times 1.053}{7}\right]$$

$$= \underline{\underline{251.892}}$$

$$F_{min} = W/b \left(1 - \frac{6e}{b}\right)$$

$$= \frac{1500}{7}\left(1 - \frac{6\times 1.053}{7}\right)$$

$$= \underline{\underline{-11.857}}$$

Reason's FOR FAILURE OF DAM

imp

1. Tensile stress developed at the face
2. over turning
3. sliding
4. excessive compression.

STABILITY OF DAM

1 → condition to avoid tension in the base of the dam

$$f_b \leq f_d$$

$$z = \frac{I}{y}$$

$$P = W$$

$$f_d = \frac{V}{A} \ (P/A)$$

$$\frac{M}{Z} \leq \frac{P}{A}$$

$$\frac{P.e}{\frac{I_0}{y}} \leq \frac{P}{A}$$

$$\frac{P.e}{\left(\frac{bd^2}{6}\right)} \leq \frac{P}{b.d}$$

$$\frac{W.e}{\frac{bd^2}{6}} \leq \frac{P}{b.d}$$

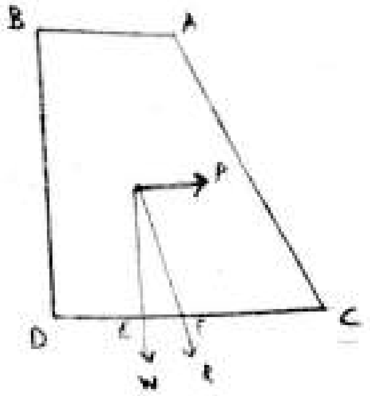
$$\frac{6We}{d} \leq W$$

$$e \leq d/6 \text{ or } b/6$$

Middle third rule

The resultant must pass through the middle third of base width to avoid tension.

2 condition to avoid overturning



for eqm

overturning moment $= P \times h/3$

Restoring Moment $= W \times EF$

For stability of dam $= P \times h/3 = W \times EF$ — ①

Dam has a tendancy to overturn about C,

overturning moment at C $= P \times h/3$

Restoring moment at C $= W \times EC$

$P \times h/3 = W \times EC$ — ②

For stability,

$W \times EC \geq W \times EF$

$\Rightarrow EC > EF$

3 conditions to prevent sliding

To prevent sliding the pressure of water should be less than frictional resistance of the dam.

μ – coefficient of friction)

$P \leq \mu W$

frictional resistance of dam

Dam will not slide if μW is grater than horizontal water pressure.

conditions to prevent crushing at the base

maximum compressible stress should be less than permissible compressive stress

$f_{max} < P_c$

MINIMUM BASE WIDTH OF DAM

i) $e = \frac{b}{6}$

ii) $P = \mu W$ – sliding

iii)

Retaining Walls

The retailed material (soil or earth)

To retail earth on one side

It act as a lateral support to the vertical slopes

Angle of repose

Natural slope of the material with the lo to make with the horizontal pressure exerted by the retained material on the - retaining wall.

Due to Pa, tendency for slide away on it's base.

Pa act on retaining wall in the

Passive earth pressure (p

* The pressure exerted on the retained material ~~on~~ when the retaining wall moves the retained material

* Due to Pp, retained material get's compressed It is only a theoretical consept and rarly occures in actual practice.

Weep holes

→ To drain water.

→ It along air in to the cavity and equalize pressure

Rankine's formula for earth pressure

$$Pa = ka \cdot \gamma h$$

$$ka = \frac{1 - \sin\phi}{1 + \sin\phi}$$

ka → coifficient of active earth pressure

γ = unit ~~of~~ weight of back fill

ϕ → Angle of internal friction.

⑤ The following perticles relate to a retaining wall or trapezoidal section having earth phase and containing earth up to top.

Not important

Height of the wall = 12m

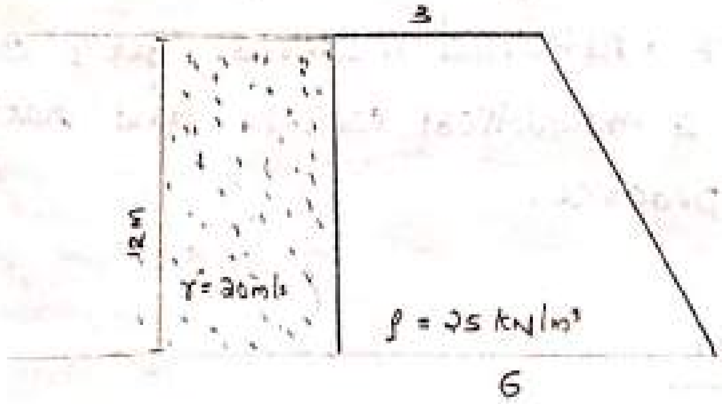
Base width = 6m

Top width = 3m

Angle of reforce = 30°

It's specific weight of soil = 20 kN/m³.

" " of wall material = 25 kN/m³



① weight ~~of~~ and retaining wall / unit length - ρA

$$\rho = \left(\frac{a+b}{2}\right) H$$

$$= 25 \times \left(\frac{3+6}{2}\right) 12$$

$$= \underline{1350 \text{ kN}}$$

② Horizondal thrust, $P = \frac{wh^2}{2} ka$

$$= \frac{wh^2}{2} \times \frac{1-\sin\phi}{1+\sin\phi}$$

$$= \frac{20 \times 12^2}{2} \times \frac{1-\sin 30}{1+\sin 30}$$

$$= \underline{480 \text{ kN}}$$

③ $R = \sqrt{W^2 + P^2}$

$= \sqrt{1350^2 + 480^2} = 1432.8$ kN

④ eccentricity = DF − b/2

$= DE + EF - b/2$

$DE = \frac{a^2 + ab + b^2}{3(a+b)}$ $EF = \frac{P}{W} \times \frac{h}{3}$

$= 2.33$ $= 1.42$

$e = 2.33 + 1.42 - 6/2$

$= 0.75\,m$

$f_{max} = \cancel{P/A + M/Z}$ $\frac{W}{b}\left(1 + \frac{6e}{b}\right)$

$= \frac{1350}{6}\left(1 + \frac{6 \times 0.75}{6}\right)$

$= 89.75\ kN/m^2$

$f_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$

$= \frac{1350}{6}\left(1 - \frac{6 \times 0.75}{6}\right)$

$= 56.27\ kN/m^2$

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MODULE - 3

DEFLECTION OF BEAMS

Design of Beam

These are Design two criterias.

* Strength :-
* Stiffness.-

Strength

→ Resist Bending and SF developed in the beam

→ cross sectional area

Stiffness

→ Beam should be stiff enough to resist deformation

→ Don't Deflect more than permissible limit

Strength

→ Resistance to failure

→ primary design criteria

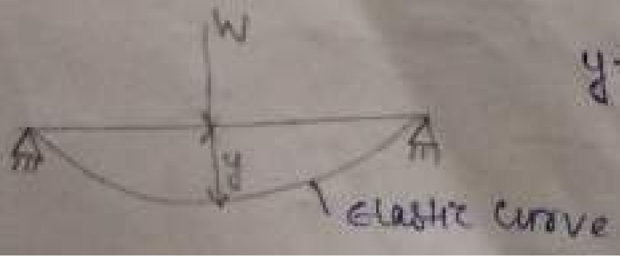
→ Its a material property

Stiffness

→ Resistance to deformations.

→ Secondary design parameter.

Note



y= deflection.

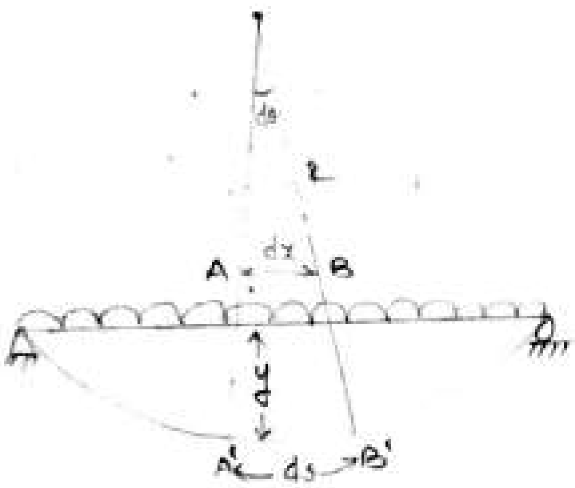
curvature of beam

Rate of change of slope with respect to length

Differential Equation for flexure (bending)

We have

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$



$$\tan\theta = \frac{dy}{dx} \Rightarrow \theta$$

$$ds = R \cdot d\theta$$

$$\frac{1}{R} = \frac{d\theta}{ds}$$

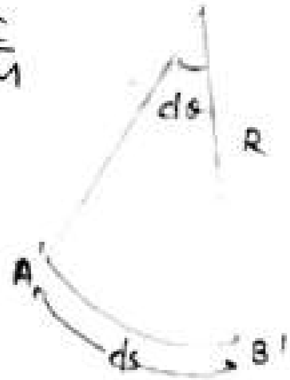
for small length, $ds \approx dx$

$$\therefore \frac{1}{R} = \frac{d\theta}{dx}$$

$$= \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d^2y}{dx^2}$$

$$R = \frac{EI}{M}$$



Replace R with $\frac{EI}{M}$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$ → Differential eqn for flexure.

METHODS OF DETERMINIG SLOPES AND DEFLECTION

* conjugate beam method

Double Integration

from Differential eqn for flexure

we have

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

integrating the Differential eqn once, we get the slope of the beam deflection.

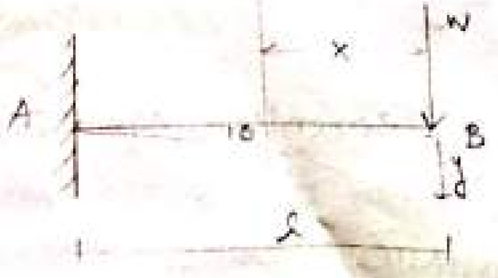
$$\text{Slope } \theta = \int \frac{d^2y}{dx^2} = \frac{dy}{dx}$$

$$= \int \frac{M}{EI}$$

Double Integrating the Differential eqn, we get deflection (y)

$$\int\int \frac{d^2y}{dx^2} = \int \frac{dy}{dx} = y$$

cantilever with a point load



$M = -wx$

applying of eqn, $\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{-wx}{EI}$

$$\int \frac{d^2y}{dx^2} = \int \frac{-wx}{EI} dx$$

$$\frac{dy}{dx} = \frac{-1}{EI}\left[\frac{wx^2}{2} + c_1\right] \quad — ①$$

At fixed end A, $\frac{dy}{dx} = 0 \quad x = l$

substitute in ①

$$0 = \frac{-1}{EI}\left[\frac{wl^2}{2} + c_1\right]$$

$$c_1 = -\frac{wl^2}{2}$$

substitute c_1 in ①,

$$\boxed{\frac{dy}{dx} = \frac{-1}{EI}\left[\frac{wx^2}{2} - \frac{wl^2}{2}\right]}$$

This is the required eqn for the slope at any section.

$$\int \frac{dy}{dx} = \int \frac{-1}{EI}\left[\frac{wx^2}{2} - \frac{wl^2}{2}\right]$$

$$y = \frac{-1}{EI}\left[\int \frac{wx^2}{2}\cdot dx - \int \frac{wl^2}{2}\cdot dx\right]$$

$$= \frac{-1}{EI}\left[\frac{wx^3}{2\times 3} - \cancel{\frac{wl^3}{2\times 3}}\ \frac{wl^2}{2}\cdot x + c_2\right] \quad — ②$$

$$= \cancel{\frac{-1}{EI}} \left[\cancel{\frac{wx^3}{6}} - \cancel{\frac{wl^3}{6}}\right] \cancel{+c}$$

At fixed end A, $y = 0 \quad x = l$

$$0 = \frac{-1}{EI}\left[\frac{wl^3}{6} - \frac{wl^2}{2}\cdot l + c_2\right]$$

$$= \frac{-1}{EI}\left[\frac{wl^3}{6} - \frac{wl^3}{2} + c_2\right]$$

$$= \frac{-1}{EI}\left[-\frac{wl^3}{3} + C_2\right]$$

$$\frac{wl^3}{6} - \frac{wl^3}{2}$$

$$= \frac{2wl^3 - 6wl^3}{6\times 2}$$

$$= -\frac{4wl^3}{12}$$

$$= -\frac{wl^3}{3}$$

$$C_2 = \frac{wl^3}{3}$$

substitute in ②,

$$y = \frac{-1}{EI}\left[\frac{wx^3}{6} - \frac{wl^2x}{2} + \frac{wl^3}{3}\right]$$

This is the required eqn for deflection at any point.

Maximum Slope

maximum slop at $x = 0$

$$\frac{dy}{dx} = \frac{1}{EI}\left[-\frac{wx^2}{2} + \frac{wl^2}{2}\right]$$

$$= \frac{-1}{EI}\left[\frac{wx^2}{2} - \frac{wl^2}{2}\right]$$

$$= \frac{-1}{EI} \times \left[0 - \frac{wl^2}{2}\right]$$

$$= \frac{wl^2}{2EI} \text{ radians}$$

Maximum Deflection

maximum deflection will be at $x = 0$

$$\cancel{y = \frac{-1}{EI}\left[\frac{wx^2}{2} + \frac{wl^2}{2}\right]}$$

$$y = \frac{-1}{EI}\left[\frac{wx^3}{6} - \frac{wl^2x}{2} + \frac{wl^3}{3}\right]$$

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$$y = \frac{-1}{EI}\left[0 - 0 + \frac{wl^3}{3}\right]$$

$$= -\frac{wl^3}{3EI}$$ —ve indicate downward deflection.

cantilever with a point load not at the end

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\theta_B = \frac{dy}{dx}$$

$$\theta_B = \frac{wl^2}{2EI}$$

$$y_B = \frac{-wl^3}{3EI}$$

small an.

slope at B

$$\theta_C = \theta_B = \frac{wl^2}{2EI}$$

Deflection at B

$$y_B = y_C + DE$$

DE = ?

$$\tan\theta = \theta_B = \frac{DE}{(l - l_1)}$$

$$\Rightarrow DE = \theta_B\,(l - l_1)$$

$$y_B = \frac{-wl^3}{3EI} + \theta_B\,(l - l_1)$$

$$= -\frac{wl_1^3}{3EI} + \frac{wl^2}{2EI}\,(l - l_1)$$

cantilever with u.d.l

From differential eqn,

$$M = -wx \cdot \frac{x}{2}$$

$$= \frac{-wx^2}{2}$$

$$\frac{d^2y}{dx^2} = -\frac{wx^2}{2EI}$$

$$\frac{dy}{dx} = \int -\frac{wx^2}{2EI} \cdot dx$$

$$= \frac{1}{EI}\int -\frac{wx^2}{2}\, dx$$

$$= \frac{1}{EI}\left[-\frac{wx^3}{6} + C_1\right] \quad \text{—①}$$

At fixed end, $\frac{dy}{dx} = 0 \quad x = l$

$$0 = \frac{1}{EI}\left[-\frac{wl^3}{6} + C_1\right]$$

$$C_1 = \frac{wl^3}{6}$$

substitute eq C_1 in ①

$$\frac{dy}{dx} = \frac{1}{EI}\left[-\frac{wx^3}{6} + \frac{wl^3}{6}\right]$$

$$\int \frac{dy}{dx} = \int \frac{1}{EI}\left[-\frac{wx^3}{6} \cdot dx - \frac{wl^3}{6}\right]$$

$$= \frac{1}{EI}\left[\int -\frac{wx^3}{6}\, dx - \int \frac{wl^3}{6}\right]$$

maximum slope

$\left(\frac{dy}{dx}\right)$ at $x = 0$

$$\left(\frac{dy}{dx}\right)_{max} = \frac{1}{EI}\left[0 + \frac{wl^3}{6}\right]$$

Deflection

Integrating ②, we get y

$$y = \int \frac{1}{EI}\left[-\frac{wx^3}{6} + \frac{wl^3}{6}\right]$$

$$= \frac{1}{EI}\left[\int -\frac{wx^3}{6} + \int \frac{wl^3}{6}\right] dx$$

$$= \frac{1}{EI}\left[\frac{-wx^4}{4\times 6} + \frac{wl^3}{6}\times x\right]$$

$$= \frac{1}{EI}\left[\frac{-wx^4}{24} + \frac{wl^3}{6}\times x\right]$$

$$= \frac{1}{EI}\left[\frac{-wx^4}{24} + \frac{wl^3x}{6} + C\right]$$

At fixed end A, y = 0 x = l

$$0 = \frac{1}{EI}\left[-\frac{wl^4}{24} + \frac{wl^3 l}{6} + C_2\right]$$

$$0 = \frac{1}{EI}\left[-\frac{wl^4}{24} + \frac{wl^4}{6} + C_2\right]$$

$$0 = \frac{1}{EI}\left[-\frac{wl^4}{24} + \frac{wl^4}{6} + C_2\right]$$

$$= \frac{1}{EI}\left[\frac{wl^4}{8} + C_2\right]$$

$$-\frac{wl^4}{24} + \frac{wl^4}{6}$$

$$\frac{-wl^4\times 6 + 24wl^4}{24\times 6}$$

$$= \frac{18wl^4}{144}$$

$$\frac{-1}{24} + \frac{1}{6}$$

$C_2 = -\frac{wl^4}{8}$, substitute in ④

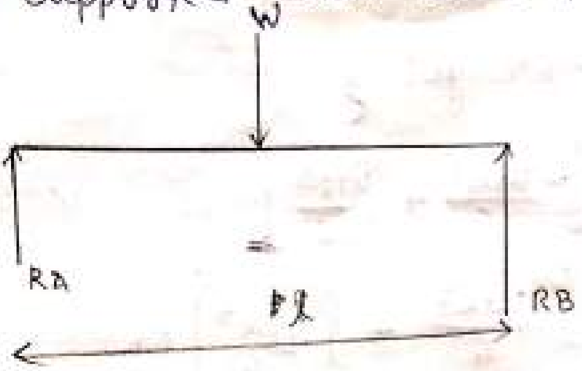
$$④ \rightarrow y = \frac{1}{EI}\left[-\frac{wx^4}{24} + \frac{wx^3 l}{6} - \frac{wl^4}{8}\right]$$

Max. Deflection

at x = 0

$$y = \frac{1}{EI}\left[0 + 0 - \frac{wl^4}{8}\right] = \frac{-wl^4}{8EI}$$

Simply Supported beam with point load at centre



$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$R_A + R_B = W$

$\Sigma M = 0$

$R_A \times 0 + W \times l/2 + R_B \times l = 0$

$Wl/2 + R_B l = 0$

$\frac{Wl}{2} = R_B l$

$R_B = W/2$

$R_A + W/2 = W$

$R_A = W/2 - W$

$= W/2$

$$\frac{d^2y}{dx^2} = \frac{Wx}{2EI}$$

$$\frac{dy}{dx} = \frac{1}{EI} \int \frac{Wx}{2} dx$$

$$= \frac{1}{EI}\left[\frac{Wx^2}{4} + C_1\right]$$

At fixed end, $\frac{dy}{dx} = 0$ $x = l/2$

$$0 = \frac{1}{EI}\left[\frac{W(l/2)^2}{4} + C_1\right] = \left(\frac{W(l/2)^2}{4} + C_1\right)$$

$(l/2)^2 = l^2/4$

$\frac{Wl^2}{4 \times 4}$

$\cancel{C_1 = \frac{Wl^2}{4 \cdot 16}}$ $C_1 = \frac{Wl^2}{16}$

Substitute c_1 in ①

$$\frac{dy}{dx} = \frac{1}{EI}\left[\frac{wx^2}{4} - \frac{wl^2}{16}\right]$$

maximum slope

At $x=0$

$$\frac{dy}{dx} = \frac{1}{EI}\left[-\frac{wl^2}{16}\right] = \frac{-wl^2}{16EI}$$

maximum deflection will be At $x=0$

Integrating ②, we get y,

$$y = \int \frac{1}{EI}\left(\frac{wx^2}{4} - \frac{wl^2}{16}\right) \times \int 1\,dx$$

$$= \frac{1}{EI}\left[\frac{wx^3}{12} - \frac{wl^2x}{16} + c_2\right] \quad — ③$$

At fixed and $\frac{dy}{dx} = 0$ $x = 0$ in eqn no ③

$$0 = \frac{1}{EI}\left(\cancel{\frac{wl^3}{12} - \frac{wl^2}{16} + c_2}\right) \quad [0 - 0 + c_2]$$

$$\cancel{= \frac{1}{EI}\left[\frac{wl^3}{48} + c_2\right]}$$

$$\cancel{c_2 = \frac{wl^3}{48}} \qquad c_2 = 0$$

Substitute in equation no. 4

$$y = \frac{1}{EI}\left[\frac{wx^3}{12} - \frac{wl^2x}{16} - \frac{wl^3}{48}\right]$$

max deflection

At $x = 0$

$$y = \frac{1}{EI}\left[0 - 0 - \frac{wl^3}{48}\right]$$

$$y = -\frac{wl^3}{48}$$

Maximum Deflection

At $x = l/2$ y_{max}

$$y = \frac{1}{EI}\left[\frac{wx^3}{12} - \frac{wl^2x}{16}\right]$$

$$y_{max} = \frac{1}{EI}\left[\frac{w(l/2)^3}{12} - \frac{wl^2 \cdot l/2}{16}\right]$$

$$= \frac{1}{EI}\left[\frac{wl^3}{8\times 12} - \frac{wl^3}{32}\right]$$

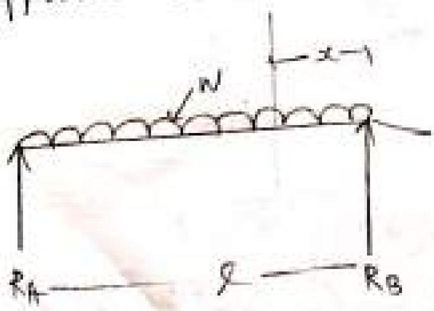
$$= \frac{1}{EI}\left[\frac{wl^3}{96} - \frac{wl^3}{32}\right]$$

$$= \frac{1}{EI}\;\frac{32\,wl^3 - 96\,wl^3}{96\times 32}$$

$$= \frac{1}{EI}\left[-\frac{wl^3}{48}\right]$$

$$= \frac{-wl^3}{48EI}$$

Simple supported beam with u.d.l



$$\frac{wl}{2} + R_B = wl$$

$$R_A + \frac{wl}{2} = wl$$

$$R_A = wl - \frac{wl}{2}$$

$$= \frac{wl \times 2 - wl}{2}$$

$$R_A + R_B = wl$$

$$\Sigma M_A = 0 \quad wl \cdot l/2 = R_B \cdot l$$

$$R_B = \frac{wl}{2}$$

$$R_A = \frac{wl}{2}$$

$$M_x = -wx \cdot x/2 + \frac{wl}{2} \cdot x = \frac{wlx}{2} - \frac{wx^2}{2}$$

$$\frac{d^2y}{dx^2} = \frac{1}{EI}\left[\frac{wlx}{2} - \frac{wx^2}{2}\right]$$

$$\frac{dy}{dx} = \frac{1}{EI}\left[\frac{wlx^2}{4} - \frac{wx^3}{6} + C_1\right] \quad \text{—— (1)}$$

At $x = l/2$, $dy/dx = 0$

$$0 = \frac{1}{EI}\left[\frac{wl(l/2)^2}{4} - \frac{w(l/2)^3}{6} + C_1\right]$$

$$= \frac{1}{EI}\left[\frac{wl^3}{16} - \frac{wl^3}{48} + C_1\right]$$

$$= \frac{1}{EI}\left[-\frac{wl^3}{24}\right]$$

$$C_1 = \frac{-wl^3}{24EI} = \frac{-wl^3}{24}$$

$$y = \frac{1}{EI}\left[\frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^3}{24}\right]$$

Maximum slope

At $x = 0$

~~QB~~ ~~dθ~~

$$\theta_B = \left(dy/dx\right)_{max}$$

$$= \frac{1}{EI}\left[0 - 0 - \frac{wl^3}{24}\right]$$

$$= -\frac{wl^3}{24EI}$$

~~Maximum~~ Deflection

$$y = \frac{1}{EI} = \left[\frac{wl \cdot x^3}{3 \times 4} - \frac{wx^4}{4 \times 6} - \frac{wl^3 x}{24} + C_2\right]$$

at $x = 0$ $y = 0$

$$0 = \frac{1}{EI}\left[0 - 0 - 0 + C_2\right]$$

$C_2 = 0$

$$\therefore y = \frac{1}{EI}\left[\frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^3x}{24}\right]$$

Maximum deflection

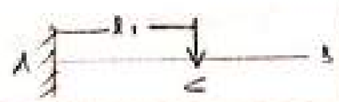
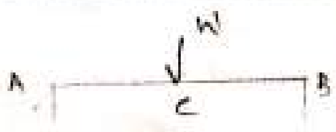
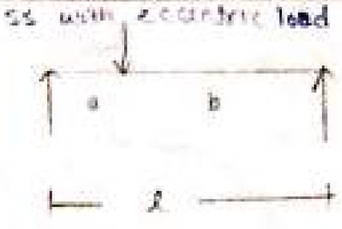
At $x = l/2$

$$y_{max} = \frac{1}{EI}\left[\frac{wl\cdot(l/2)^3}{12} - \frac{w(l/2)^4}{24} - \frac{wl^3\cdot(l/2)}{24}\right]$$

$$y_{max} = \frac{1}{EI}\left[\frac{wl^4}{8\times12} - \frac{wl^4}{16\times24} - \frac{wl^4}{48}\right]$$

$$= \frac{1}{EI}\left[\frac{wl^4}{96} - \frac{wl^4}{384} - \frac{wl^4}{48}\right]$$

$$= \frac{-5}{384}\,\frac{wl^4}{EI}$$

Loading	maximum Slope $\theta = dy/dx$	Deflection, y
	$\theta_B \; \frac{wl^2}{2EI}$	$y_B = \frac{-wl^3}{3EI}$
	$\theta_B = \theta_C = \frac{wl^2}{2EI}$	$y_B = \frac{-wl_1^3}{3EI} + \frac{wl^2}{2EI}(l-l_1)$
	$\theta_B = \frac{wl^3}{6EI}$	$y_B = \frac{-wl^4}{8EI}$
	$\theta_B = \frac{-wl^2}{16EI}$	$y_B = \frac{-wl^3}{48EI}$
	$\theta_B = \frac{-wl^3}{24EI}$	$y_B = \frac{-5}{384}\frac{wl^4}{EI}$
SS with eccentric load 	$\theta_A = \frac{-wa}{6EIL}(l^2-b^2)$ $\theta_B = \frac{-wa}{6EIL}(l^2-a^2)$	$y_C = \frac{-wa^2b^2}{3EIL}$

- A beam 5 m long simply supported at its ends - carrying a point load W at its center. If the slope at the ends of the beam is not to exceed 1°, find the deflection at the centre of beam.

$\theta_B = \dfrac{-wl^2}{16EI} = 0.0174$

$L = 5m$

$w = ?$

$E = ?$

$\theta_B = 1^\circ \times \pi/180 = 0.0174$

$\theta_B = \dfrac{-wl^2}{16EI}$ radians

$0.0174 = \dfrac{5^2}{16} \times \dfrac{-w}{EI}$

$0.0174 \times \dfrac{16}{25} = \dfrac{-w}{EI}$

$\dfrac{0.0174 \times 16}{25} = \dfrac{-w}{EI}$

$0.011 = \dfrac{-w}{EI}$

$y_c = \dfrac{-wl^3}{48EI}$

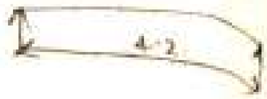
$= 0.011 \times \dfrac{5^3}{48}$

$= 0.011 \dfrac{125}{48}$

$= \dfrac{0.011 \times 125}{48}$

$= 0.0286$

* Calculate the maximum deflection of centre of Simple supported beam of span 4.2 when the maximum slope is 3°



$$Q_B = 3^\circ$$

$$= 3 \times \pi/180$$

$$= \underline{\underline{0.0523}}$$

$$= \frac{16 \times 0.0523}{4.2}$$

$$\frac{-wl^2}{16EI} = \frac{-w}{EI} \times \frac{(4.2)^2}{16}$$

$$\frac{-w}{EI} = \frac{0.0523}{1.1025}$$

$$= \underline{\underline{-0.0474}}$$

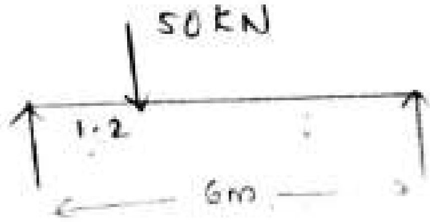
$$y_C = \frac{-wl^3}{48EI}$$

$$= 0.0474 \times \frac{(4.2)^3}{48}$$

$$= \underline{\underline{-0.0731}}$$

* A steel beam S.S over a span of 6m carries a point load of 50 kN at 1.2 m from left hand support find the position and magnitude of maximum deflection.

Take $EI = 14 \times 10^{12}\ N/mm^2$



$a = 1.2$

$b = 4.8$

$$y_c = \frac{-w a^2 b^2}{3EIL}$$

maximum deflection under point load.

$$= -\frac{\cancel{5000}\ \cancel{500 \times 1.2^2 \times 4.8^2}}{\cancel{3 \times 14 \times 10^{12} \times 6}}$$

$$= -\cancel{50\times}\quad \cancel{\times 28.04}$$

$$-\frac{5000 \times (1.2 \times 10^{-3})^2 \times (4.8 \times 10^{-3})^2}{3\cancel{6000} \times 14 \times 10^{12} \times 6000}$$

1.65

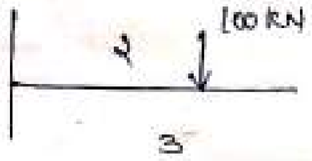
$$= -\frac{5000 \times (1200)^2 \times (4800)^2}{3 \times 14 \times 10^{12} \times 6000}$$

$$= \frac{1.65888 \times 10^{-17}}{2.52 \times 10^{-17}}$$

$= \underline{\underline{6.582}}$ mm downwards.

A cantilever beam of span 3m carries a ~~per~~ point load 100 kN At a distance of 2m from the fixed end. determine the slope and deflection at free end Take

$E = 200\ kN/mm^2 \quad I = 400 \times 10^6\ mm^2$



$W = 100\,kN$

$l = 3m$

$E = 200\ kN/mm^2$

$I = 400 \times 10^6\ mm^2$

$\theta = \frac{Wl_1^2}{2EI} = \frac{100 \times 2000^2}{2 \times 200 \times 400 \times 10^6}$

$= \underline{\underline{2.5 \times 10^3\ mm}}$

$m = \frac{wl^2}{8} = \frac{10 \times 6000}{8}$

$= \underline{\underline{45 \times 10^6}}$

$y = \frac{7 \times 1.687 \times 10^{12}}{45 \times 10^6}$

$= \underline{\underline{261333}}$

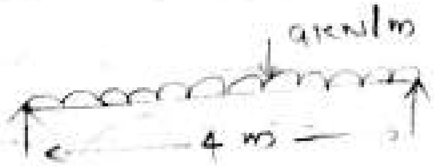
$y = \frac{-Wl_1^3}{3EI} + \frac{Wl^2}{2EI}(l - l_1)$

$= \frac{100 \times 2000^3}{3 \times 200 \times 400 \times 10^6} + \frac{100 \times 3000^2}{2 \times 200 \times 400 \times 10^6}(3000 - 2000)$

$y = \underline{\underline{2.291\ mm}}$

A beam of uniform section 200 ~~mm~~ x 300 mm is S.S at the ends. It carries a udl of 9 KN/m over the entire span ~~R~~ 4 m. Find the maximum slope and maximum deflection

$E = 1 \times 10^4 \ N/mm^2$



Area of CS = 200 x 300 mm

$I = 1 \times 10^4 \ N/mm^2$

$$\text{maximum slope} = \frac{-w l^3}{24 E I}$$

$$= \frac{-9 \times 4^3}{24 \times 1 \times 10^4 \times I}$$

$$= \frac{-9 \times 4^3}{24 \times 1 \times 10^4 \times 4.5 \times 10^{-4}}$$

$$= -\frac{576}{108}$$

$$= -5.333$$

$$I = \frac{bd^3}{12}$$

$b = 0.2$

$d = 0.3$

$$= \frac{0.2 \times (0.3)^3}{12}$$

$$= 4.5 \times 10^{-4}$$

$$\text{maximum deflection} = \frac{-5}{384} \frac{w l^4}{E I}$$

$$= \frac{-5}{384} \frac{9 \times 4^4}{1 \times 10^4 \times 4.5 \times 10^{-4}}$$

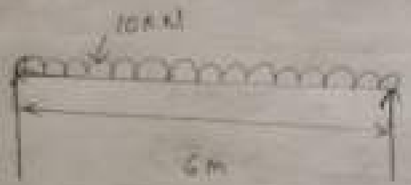
$$= -0.0130 \ \frac{2304}{4.5}$$

$$= -0.0130 \times 512$$

$$= +6.656$$

• A beam of length 6m and of uniform rectangular section is s.s at its sections it carries a udl of 10 kN per meter over the entire span. Calculate the depth and breadth required if permissible bending stress is 7 N/mm² and central deflection is not exceed 10 mm

$E = 1\times10^4$ N/mm²



d = ?

b = ?

$\frac{M}{I} = \frac{f}{y}$

$y = d/2$

$I = \frac{bd^3}{12}$

$M = \frac{wl^2}{8}$

$f = 7$ N/mm²

$$y = \frac{-5}{384}\ \frac{wl^4}{EI}$$

$$= \frac{-5}{384}\ \frac{10000\times6000^4}{1\times10^4\times I}$$

$$10 = \frac{5}{384}\ \frac{1.296\times10^{19}}{1\times10^4\times I}$$

$$I = \underline{\underline{1.684\times10^9}}$$

$$M = \frac{wl^2}{8} = \frac{10\times6000^2}{8} = 45\times10^6$$

$f = 7$ N/mm²

$$\frac{M}{I} = \frac{f}{y} \Rightarrow y = \frac{fI}{M} = \frac{7\times1.6875\times10^9}{45\times10^6} = 262.5 \text{ mm}$$

$$y = d/2 = 262.5$$

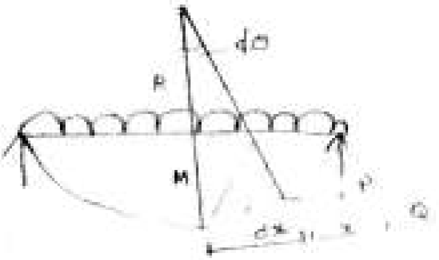
$$d = 262.5\times2 = \underline{\underline{525\text{ mm}}}$$

$$I = \frac{bd^3}{12} \quad b = \frac{12I}{d^3} = \frac{12\times1.6875\times10^9}{525^3}$$

$$= 139.94$$

$$= \underline{\underline{140\text{ mm}}}$$

Moment Area Method



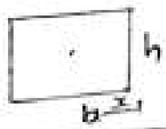
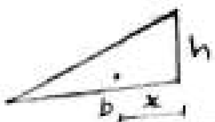
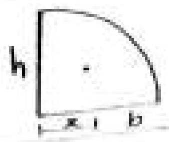
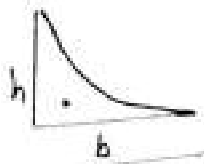
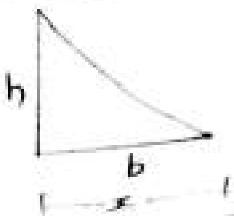
$$\frac{M}{I} = \frac{E}{R} \Rightarrow \frac{1}{R} = \frac{M}{EI}$$

$$\theta = \frac{A}{EI}$$ (Mohr's first theorem)

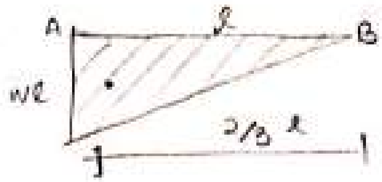
$$PQ = x\, d\theta$$
$$= x \cdot \frac{M}{EI} \cdot dx$$

$$y_{max} = \int PQ = \int x \cdot \frac{M}{EI} \cdot dx$$

$$y = \frac{A \cdot \bar{x}}{EI}$$ → Mohr's 2^{nd} theorem

FIGURE	A	$\bar{x}$
	$A = bh$	$\bar{x} = b/2$
	$A = \frac{1}{2} bh$	$\bar{x} = b/3$
	$A = \frac{2}{3} bh$	$\bar{x} = \frac{3}{8} b$
	$A = \frac{1}{3} bh$	$\bar{x} = \frac{3}{4} b$
	$A = \frac{1}{n+1} bh$	$\bar{x} = b\frac{(n+1)}{(n+2)}$

cantilever with point load at end



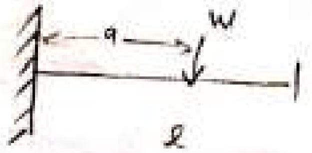
Slope $Q_B = \frac{A}{EI} = \frac{\frac{1}{2} wl \cdot l}{EI} = \frac{wl^2}{2EI}$

Deflection $y_B = \frac{A\bar{x}}{EI} = \frac{2/3\, l^3}{3EI}$

$= \frac{wl^2}{2EI} \times \frac{2}{3} l = \underline{\underline{\frac{wl^3}{3EI}}}$

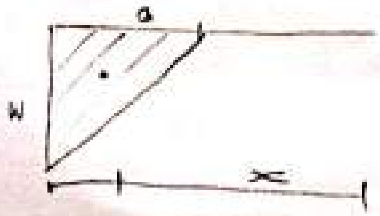
15*

cantilever with point load at end



$\bar{x} = \frac{2}{3} a + l - a$

$= l - \frac{a}{3}$



Slope $Q_B = \frac{A}{EI}$

$= \frac{\frac{1}{2} Wa \times a}{EI}$

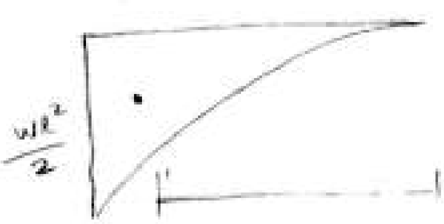
$= \frac{wa^2}{2EI}$

Deflection $y_B = \frac{A\bar{x}}{EI}$

$= \frac{wa^2}{2EI} - \left(l - \frac{a}{3}\right)$

Cantilever with udl

$1/3\,ah$

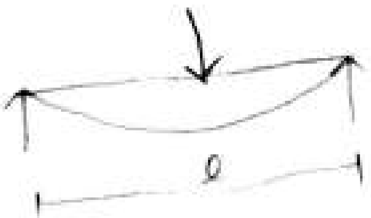


$$\text{Slop } \theta_B = \frac{A}{EI} = \frac{1/3 \times l \times \frac{wl^2}{2}}{EI} = \frac{wl^3}{6EI}$$

$$y_B = \frac{wl^3}{6EI} \times \bar{x} = \frac{wl^3}{6EI} \times 3/4 \times l$$

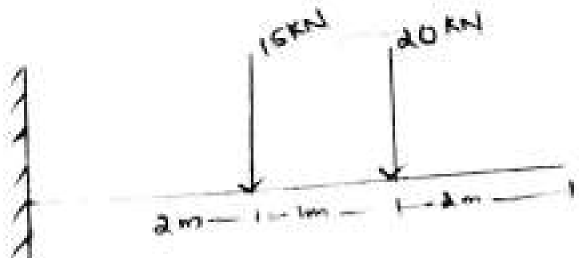
15×

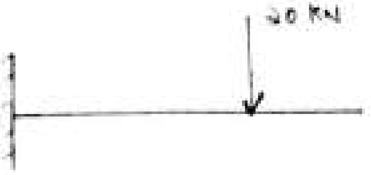
$$= \frac{wl^4}{8EI}$$

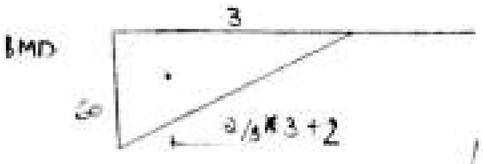


A cantilever of span 5 m carries point loads of 20 kN at 2m from the free end. And 15 kN from . using moment theorem find the maximum ~~....~~ ~~..~~ slope ad deflection

$EI = 8400$ kN







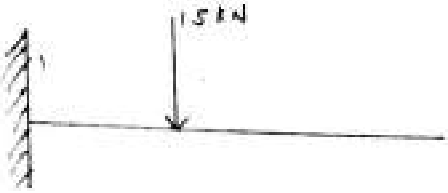
$$Q_B = Q_1 + Q_2$$

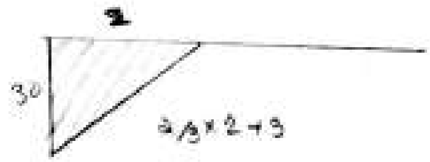
$$= \frac{A_1}{EI} + \frac{A_2}{EI}$$

$$= \frac{\frac{1}{2}\times 60\times 3 + \frac{1}{2}\times 2\times 30}{8400}$$

$$= \frac{120}{8400}$$

$$= \underline{\underline{0.0142 \text{ radians}}}$$





Deflection at free end,

$$y_B = y_1 + y_2$$

$$= \frac{A_1 \bar{x}_1}{EI} + \frac{A_2 \bar{x}_2}{EI}$$

$$x_1 = \frac{2}{3}\times 3 + 2 = 4$$

$$x_2 = \frac{2}{3}\times 2 + 3 = 4.33$$

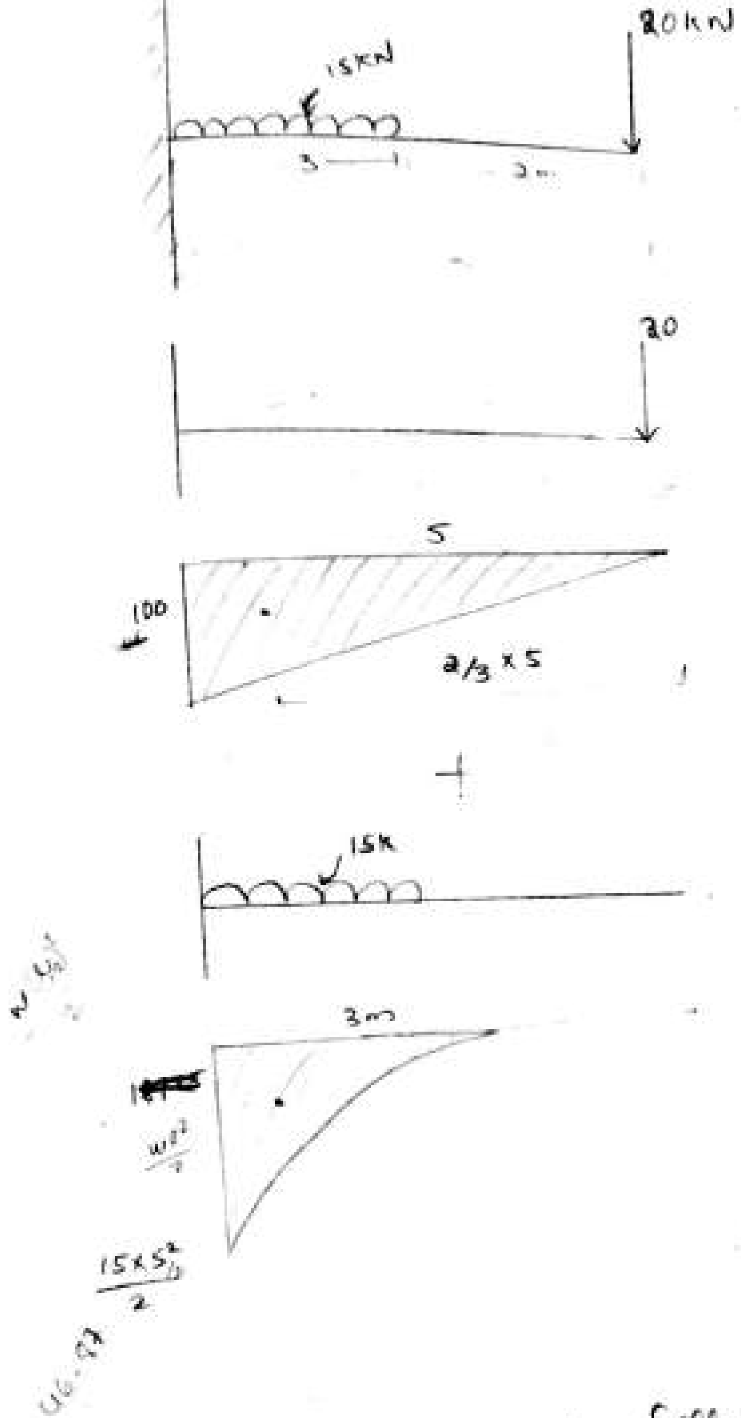
$$y_B = \frac{90\times 4 + 30\times 4.33}{3400}$$

$$= \underline{\underline{0.058}}$$

- A cantilever 5 m length carries ~~of~~ a point load of 20 kN at the free end in addition to udl of 15 kN/m. over ~~to~~ a length of 3 m from the fixed end. Determine the maximum slop in the beam.

$E = 2\times 10^5 \ N/mm^2$

$I = 3\times 10^8 \ mm^4$



Slop @ $Q_B = \frac{A_1}{EI}$

$= \frac{1/2 \times 100 \times 5}{2 \times 10^5 \times 3 \times 10^8}$

$= \frac{250}{6 \times 10^{13}}$

$= 4.166 \times 10^{-12}$

Slop $Q_A = \frac{A_2}{EI}$

$= \frac{1/3 \times 5 \times 46.87}{2 \times 10^5 \times 3 \times 10^8}$

$= \frac{78.116}{6 \times 10^{13}}$

$= 1.301 \times 10^{-12}$

Slope $Q_B + Q_A$

$= 4.166 \times 10^{-12} + 1.301 \times 10^{-12}$

$= 5.46793 \times 10^{-12}$ radin

1.388×10^{-11}

Deflection at free end

$y_B = y_1 + y_2$

$= \frac{A_1 \bar{x}_1}{EI} + \frac{A_2 \bar{x}_2}{EI}$

$= \frac{1/2 \times 100 \times 5 \times 2/3 \times 5}{2 \times 10^5 \times 3 \times 10^8} + \frac{1/3 \times 5/2 \times 46.87}{2 \times 10^5 \times 3 \times 10^8}$

$= 1.388 \times 10^{-11} + 6.509 \times 10^{-13}$

$= 1.45309 \times 10^{-11}$

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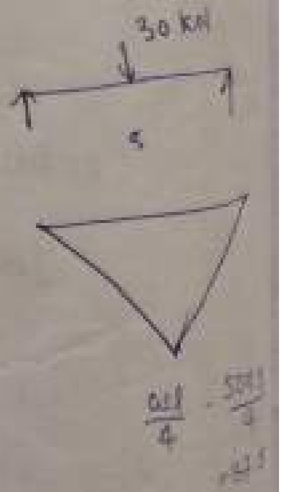
① A simply supported beam of span 5m carries a central point load of 30 kN. Find the max. deflection and slope using moment area method?

② * A SS of span of 4 m carries a udl of 10 kN/m on the entire span. If E = 210 kN/mm² and I = 8.98×10⁻ ... Find the maximum slope and deflection by using moment area method.

③ * A SS ~~of bea~~ beam of span 6m carries

ANSWERS



1) $\theta_B = \dfrac{\frac{1}{2} \times 37.5 \times 2.5}{4200}$

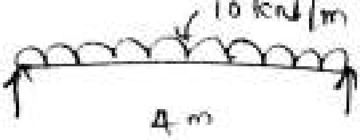
$= 0.0116$ Radians

max deflection at mid span

$y_c = \dfrac{46.8 \times \frac{2}{3} \times 2.5}{4200}$

$= 0.0182\,m$

(2)



$$Q_B = \frac{A}{EI}$$

$$= \frac{2/3\, bh}{EI}$$

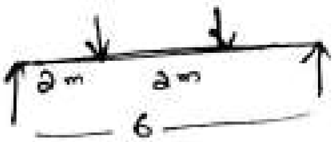
$$= \frac{2/3 \times 400 \times 8000}{210 \times 8.98 \times 10^6}$$

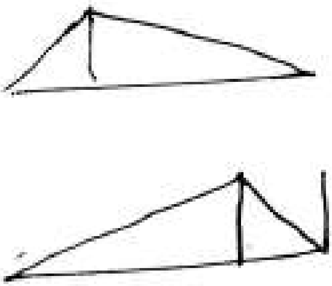
$$Q_B = \underline{\underline{0.1131 \text{ radians}}}$$

$$y_B = \frac{A\bar{x}}{EI} = \frac{2/3 \times 4000 \times 8000 \times 3/8 \times 4000}{210 \times 8.98 \times 10^6}$$

$$= \underline{\underline{169.68 \text{ mm}}}$$

(3)





$$Q_B = Q_1 + Q_2$$

$$Q_1 = \frac{1/2 \times 2 \times 60 + 1/2 \times 4 \times 120}{2000}$$

$$= \underline{\underline{0.01}}$$

$$Q_2 = 0.01 + 0.01$$

$$= \underline{\underline{0.02 \text{ rad}}}$$

$$y_B = 0.018 + 0.0188$$

$$= 0.036 \text{ m}$$

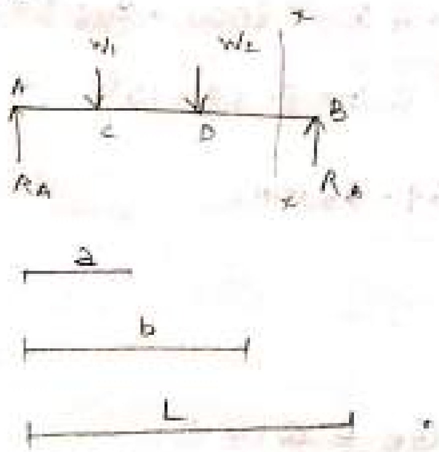
$$= \underline{\underline{36 \text{ mm}}}$$

Macauly's method

It is a simple method to find slope and deflection of beams.

It is used in case of several point loads or continuous load.

It uses a single expression to find slope and deflection.



STEPS

1 Find Reactions

2. Find the moment about the section xx

* For portion AC [x less than A] only the first term of the equation is considered

* For portion CD [x less than b/w A & B] The first two terms are considered.

* For portion DB [x greater than B] all the three terms are considered.

3 Find the slop and deflection from the above eqn using $d^2y/dx^2 = \frac{M}{EI}$

$$EI \cdot \frac{d^2y}{dx^2} = M_x$$

NOTE

when negative terms comes inside the bracket eliminate it

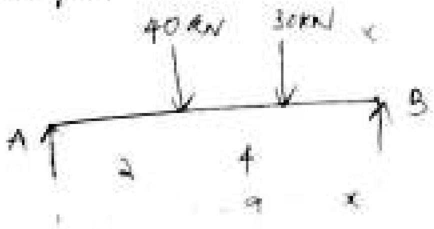
$$EI \cdot \frac{d^2y}{dx^2} = R_A x - W_1 (x-a) - W_2 (x-b)$$

Integrating.

$$EI \, dy/dx = R_A \, x^2/2 + C_1 - W_1 \frac{(x-a)^2}{2} - \frac{W_2 (x-b)^2}{2}$$

$$EI \, y = \frac{R_A}{2} \cdot \frac{x^3}{3} + C_1 x + C_2 - \frac{W_1}{2} \frac{(x-a)^3}{3} - W_2/2 \frac{(x-b)^3}{3}$$

- A simply supported beam of span 9 m is loaded as shown. If $E = 200 \, kN/mm^2$ and $I = 16 \times 10^8 \, mm^4$ using macauly's method, calculate
  a) Deflection under the load
  b) Maximum deflection in the beam
  c) slope at A



Find the Reactions

$$R_A + R_B = 40 + 30 = 70 kN$$

$$R_A \times 0 + 40 \times 2 + 30 \times - R_B \times 9$$

$$= 80 + 180 + R_B 9$$

$$0 + 260 + R_B 9$$

$9 RB = 260$

$RB = \frac{260}{9}$

$= \underline{\underline{28.88}}$

$RA + RB = 70 kN$

$RA + 28.88 = 70 kN$

$RA = 70 - 28.88 = \underline{\underline{41.11}}$

Moment about section xx:

$M_x = RA \cdot x - W_1 (x-2) - W_2 (x-6)$

$= 41.2 \times x - 40 (x-2) - 30 (x-6)$

Equation for slope and deflection

$EI \frac{d^2y}{dx^2} = M_x$

$EI \frac{d^2y}{dx^2} = 41.2x - 40(x-2) - 30(x-6)$

Integrating

$EI . y = \frac{41.2}{2} \frac{x^3}{3} + c_1 x + c_2 - \frac{40}{2} \frac{(x-2)^3}{3} - 30/2 \frac{(x-}{3}$

Eqn for deflection $= 6.86 x^2 - 6.67 (x-2)^3 - 5 (x-6)^3 -$

Applying End conditions,

A, $x = 0$ $y = 0$

Substituting in (2)

$c_2 = 0$

Substitute in (2)

$c_1 = 0$ $x = 9$ $y = 0$

$= 6.86 \times (9)^3 - 6.67 (9-2)^3 - 5 (9-6)^3$

$= 6.86 \times (9)^2 + c_1 9 + c_2 - 6.67 (9-2)^3 - 5 (9-6$

$D = 6.86 \times 9^3 + 0 - 6.67 \times (9-2)^3 - 5\ (9-6)^3$

$C_1 = \underline{\underline{-286}}$

Revised equation for slope and deflection.

$\therefore$ eqn for slope is

$$EI = \frac{-dy}{dx} = \frac{41.2x^2 - 286}{2} - \frac{40\ (x-2)^2}{2} - \frac{30\ (x-6)^2}{2}$$

equation in deflection

$$EI \cdot y = 6.86\, x^2 - 286\, x + 0 - 6.67\ (x-2)^3 - 5\ (x-6)^3$$

Deflection under loads, under load 40 KN $x = 2x \Rightarrow 2$

$$EI \cdot y = 6.86 \times (2)^3 - 286 \times 2 + 0 - 6.67\ (2-2^3) - 5(2-6)$$

$$= (6.86 \times 2)^3 - (286 \times 2) = \underline{\underline{517.12}}$$

$$EI = \frac{200\text{ KN}}{mm^2} \times 16 \times 10^8\ mm^2$$

$$= \frac{517.12}{200 \times 16 \times 10^8 \times 10^{-6}}$$

$$= \underline{\underline{-1.616 \times 10^{-3}}}$$

Maximum deflection in a beam

$EI\ dy/dx = 41.2\ \underline{x^2}$

Section b/w C-D

$EI\ dy/dx = 41.2\ x^2/2 + C_1 - \frac{40\ (x-2)^2}{2} - 30(x-6)$

→ Equations of static equilibrium = $\Sigma H = 0$ $\Sigma V = 0$ ΣM

No of static equilibrium equation in a plane = 3

Statically determinite beams;

Beams which can be analysed using equations of equilibrium are called statically determinate beams.

Eg :- Simply Supported Beam (cantilever)

Statically in determinate beams :

Beams which cannot be analyzed using the conditions of static equilibrium are called statically in determinate beams.

Eg : proped cantilever , Fixed Beams

continuous beams

Degree of static in determining [SI]

No of unknown forces in ~~exce~~ Excess of conditions of static equilibrium.

SI = No of unknown forces − No of equilibrium equations.

No	Beam	No of unknown forces	SI =
1	 Propped cantilever	4	4 − 3 = 1
2	 fixed beam	6	6 − 3 = 3
3	 continous beam	5	5 − 3 = 2
4	 cantilever	3	3 − 3 = 0
5 →			

Fixed Beams :- (Encaster Built in)

A beam with both ends are effectively fixed

Slope = 0
deflection = 0

shape in
deflection max

slope = 0
deflection = 0

Slope and deflection at the ends are called -
fixed end moment (FEM)

The Restraining moment (FEM) will be opposi
to that of loading moments

FEM bends the beam with convexities up
causing hogging moments and vice versa.

Advantages of fixed beams:-

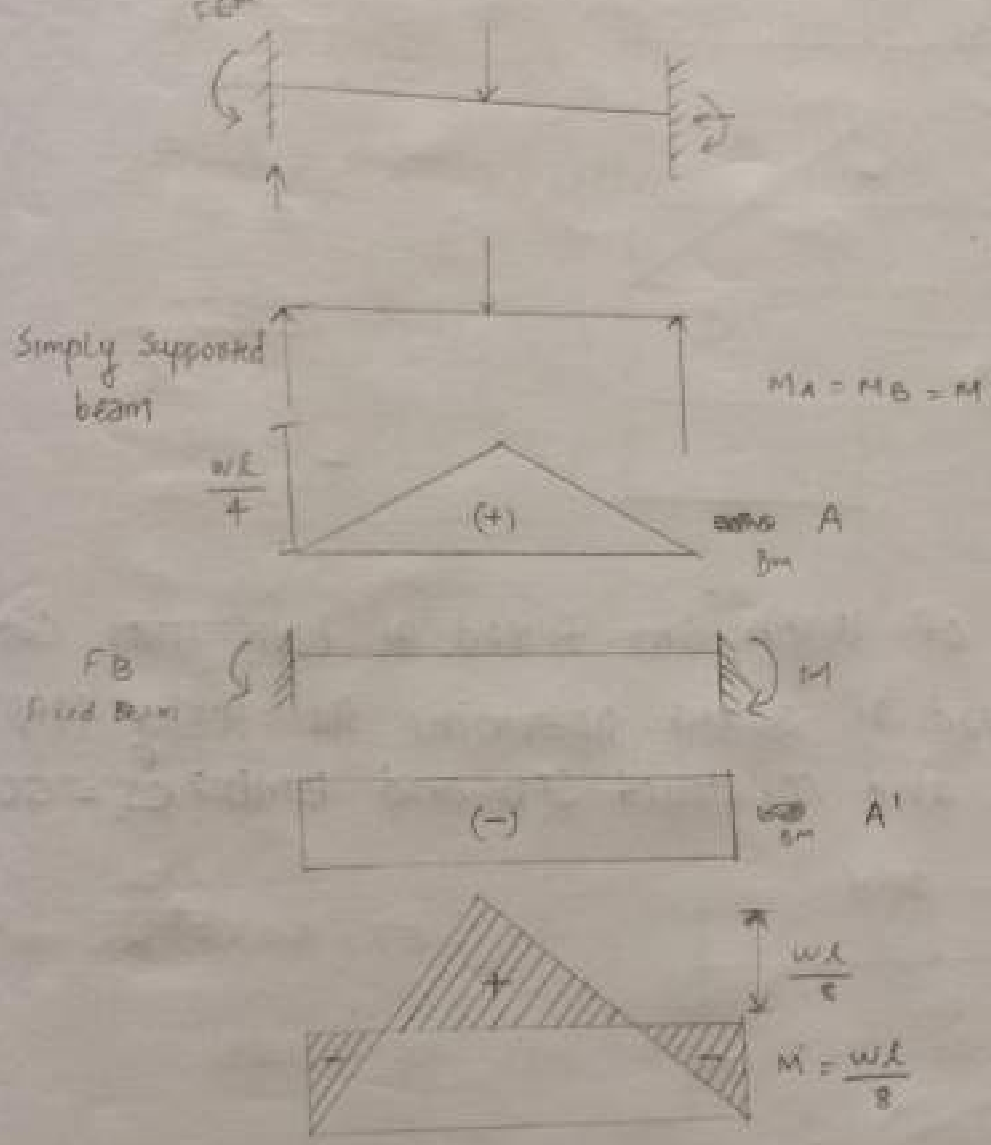
1 Bending moments are less.

2 Deflection are less. Max.m defl^n also have
value compared to other beams

3 They are stronger and stiffer.

Due to temparature changes large stresses are

* Any sinking of one support induces large stresses
* Extra care is needed in aligning the support accu
at the same level
* Frequent changes in loading especially in case of
moving loads makes the degree of fixity at the
very uncertant.



Note

Area $= A = A'$

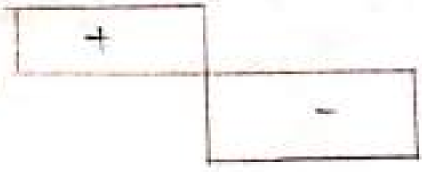
$= \frac{1}{2} \times \frac{wl}{4} \times l = Ml$

$M = \frac{wl}{8}$

① 

$\frac{wl}{8} - \frac{wl}{8}$

Hogg mathrsm BMD 

SFD 

② A Beam of length 6m. fixed at both ends carries a central load of 30 kN. Determine the fixing couples at the ends and construct SFD and BMD? $EI = 50\times10^9$ kN

imp

directly

$EI = 50\times10^9$ kN·mm²

M = ?

BMD = ?

SFD = ?

$\frac{wl}{4} = \frac{30\times6}{4} = 45$ kN.m

$M = \frac{wl^2}{8}$

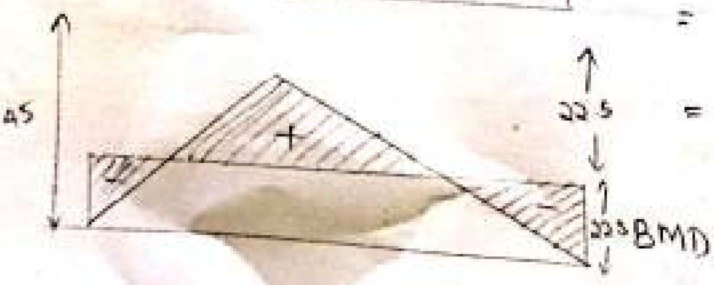
$= \frac{30\times6^2}{8}$

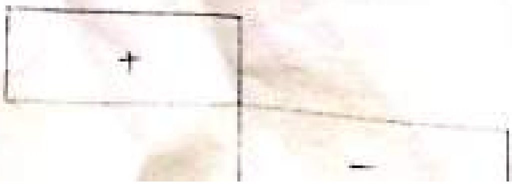
$= 22.5$ kNm

A = A'

$\frac{1}{2}\times\frac{wl}{4}\times l = M\times l$

$M = \frac{wl}{8}$



SFD 

$\frac{w}{2} = \frac{30}{2} = 15$

(b) fixed beam with udl

$M = \frac{wl^2}{8}$

$M = \frac{wl^2}{12}$

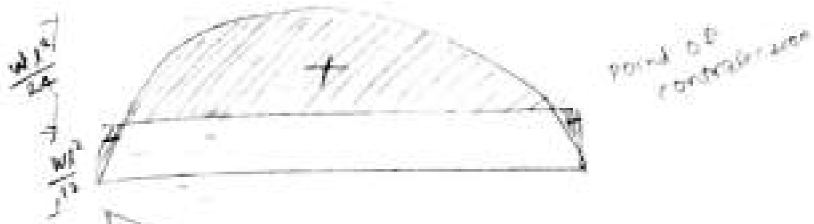
$= \frac{2}{3} \times \frac{wl^2}{8} \times l$

$= \frac{2}{3} \times \frac{wl^2}{8} \times l = M \times l$

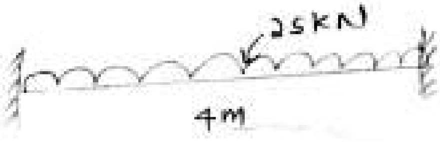
$M = \frac{wl^2}{12}$

$= \frac{2wl^2}{24}$

$M = \frac{wl^2}{12}$



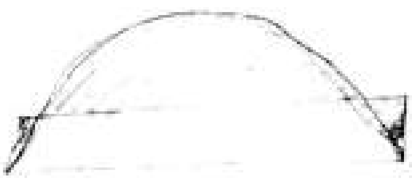
- A fixed beam of span 4 m carries an udl of 25 kN per m. over the entired length. Draw the SFD and BMD.



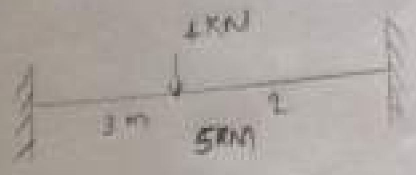
$M = \frac{wl^2}{8} = \frac{25 \times 4^2}{8} = 50$ kN.m

$M = \frac{wl^2}{12} = \frac{25 \times 4^2}{12} = \cancel{4.166}$ 33.3 kN.m

$SFD = \frac{wl}{2} = \frac{25 \times 4}{2} = 50$



A Beam of 5m length carries a consentrated 4 KN at a distance of 3m from the left sup determine fixed end moment



$$M_A = \frac{wa \cdot b^2}{l^2}$$

$$= \frac{4 \times 3 \cdot 2^2}{5^2}$$

$$= \underline{\underline{1.92}}$$

$$M_B = \frac{wb \cdot a^2}{l^2}$$

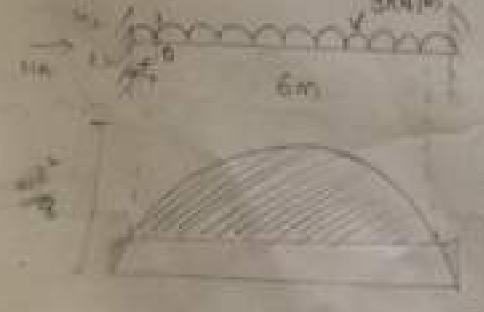
$$= \frac{4 \times 2 \times 3^2}{5^2}$$

$$= \underline{\underline{2.88}}$$

A Beam of 6m span has its ends built in and carries a udl of 3 KN/m. find

1. maximum BM
2. SFD and BMD
3. position of point of contraflexure

$EI = 20 \times 10^8$ KN. mm² (structural rigidity)



$$M = \frac{wl^2}{8} = \frac{3 \times 6^2}{8} = \underline{\underline{13.5}} \text{ KN.m}$$

BMD: $M = \frac{wl^2}{12} = \frac{3 \times 6^2}{12} = \underline{\underline{9}}$ KN.M

$$SFD = \frac{wl}{2} = \frac{3 \times 6}{2} = 9$$

$$RA \cdot x - MA - \frac{Wx \cdot x}{2} = 0$$

$\Sigma M_D = 0$

$$RA \cdot x - MA - \frac{Wx \cdot x}{2} = 0$$

$$9 \cdot x - 9 - \frac{3x^2}{2} = 0$$

$$\frac{3x^2}{2} - 9x + 9 = 0$$

$a = \frac{3x^2}{2}$

$b = 9x$

$c = 9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-9 \pm \sqrt{81 - 4 \times 3/2 \times 9}}{2 \times 3/2}$$

$$= -9 \pm \sqrt{81 -}$$

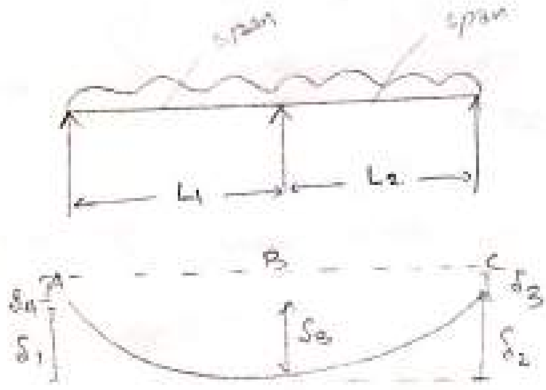
- A Beam of length 6m is fixed at both ends carries a consentratted load for 40 kN at its — middle. determine the fixing cuple (M_A, M_B) at the ends?

30 kN/m

- An encastered beam 2m long is subjected to an udl of 20 kN/m. Determine SFD and BMD? The maximum +ve, −ve moments?

3.3
−6.7

Analysis of continous Beams



Clapeyron's theorem for three moments.

$$M_A \cdot \frac{L_1}{E_1 I_1} + 2M_B\left(\frac{L_1}{E_1 I_1} + \frac{L_2}{E_2 I_2}\right) + M_C \frac{L_2}{E_2 I_2} +$$

$$\frac{6 A_1 \bar{x}_1}{E_1 I_1 L_1} + \frac{6 A_2 \bar{x}_2}{E_2 I_2 L_2} + 6\left(\frac{\delta_1}{L_1} + \frac{\delta_2}{L_2}\right) = 0$$

M_A, M_B & M_C $\longrightarrow$ moments

L_1, L_2 $\longrightarrow$ span length

$E_1 I_1$ $\longrightarrow$ ~~Re~~ flexural rigidity

A_1 $\longrightarrow$ Area of BMD_1

A_2 $\longrightarrow$ Area of BMD_2

$\bar{x}_1$, $\bar{x}_2$ $\rightarrow$ centroidal distance.

δ_1, δ_2 $\rightarrow$ Settlements with respect to B

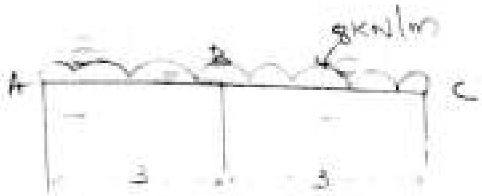
Case

i) $E_1 I_1 = E_2 I_2 = EI$

ii) No settlement (=0)

$$M_A \cdot L_1 + 2M_B(L_1+L_2) + M_C L_2 + \frac{6A_1\bar{x}_1}{L_1} + \frac{6A_2\bar{x}_2}{L_2} = 0$$

- A continues beam ABC with s end support having spans AB = BC = 3m. It is applied with a udl of 8kN/m for the entire length calculates the support moments.

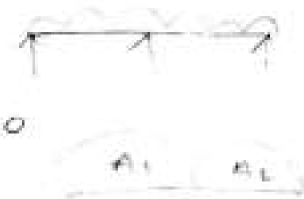


Note

If the two span continues beam has end suports which are simply supported then $M_A = \cancel{M_B} = M_C = 0$

$$M_A \cdot L_1 + 2M_B(L_1+L_2) + M_C L_2 + \frac{6A_1\bar{x}_1}{L_1} + \frac{6A_2\bar{x}_2}{L_2} = 0$$

$M_A = M_C = 0$



$$0 + 2M_B(6) + 0 + \frac{6\times18\times1.5}{3} + \frac{6\times18\times1.5}{3} = 0$$

$$0 + 2M_B(6) + 0 + 54 + 54 = 0$$

$A_1 = 2/3 \times \frac{wl^2}{8} \times l$

$$12M_B + \cancel{5040}\ 108 = 0$$

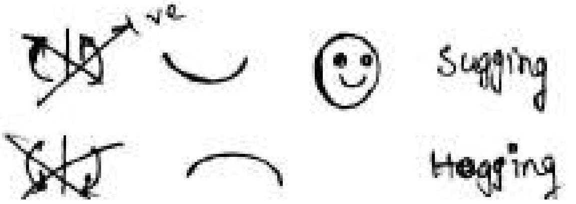
$= 2/3 \times \frac{8\times3^2\times3}{8}$

$= \underline{18}$

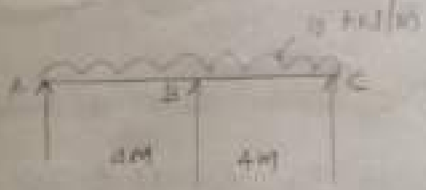
$$M_B = \frac{-108}{12}$$

$= \underline{-9}$ (Hogging moment)

$\bar{x}_1 = \bar{x}_2 = L/2$



A con



find. Support moments ?

$$M_A \cdot L_1 + 2M_B(L_1+L_2) + M_C L_2 + \frac{6A_1\bar{x}_1}{L_1} + \frac{6A_2\bar{x}_2}{L_2}$$

$M_A = M_C = 0$

$$0 + 2M_B(8) + 0 + \frac{6\times 53.66\times 2}{4} + \frac{6\times 53.66\times 2}{4}$$

$= \cancel{2M_B(8) + 127.98 + 127.98 = 0}$

$= \cancel{16 M_B + 255.96 = 0}$

$= 16 M_B + 160.98 + 160.98$

$= 16 M_B = -321.96$

$$M_B = -\frac{321.96}{16}$$

$= \underline{20.12}$

$A_1 = 2/3 \times \frac{wl^2}{8}$

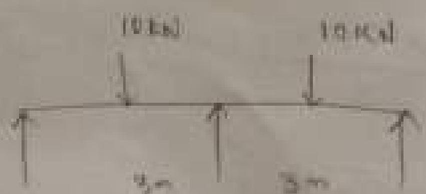
$= 2/3 \times 10...$

$x_1 = x_2$

$= l/2$

$= 4/2$

$= 2$



$A_1 = A_2 = 1/2 \times \frac{wl}{4}$

$= 1/2 \times ...$

$= \frac{45}{6}$

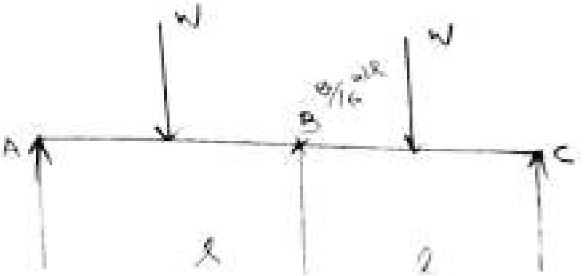
$x_1 = x_2 = ...$

A two span continues span has two eql span with a point load w at the middle of each span

15 mark

① Find the fixed end moments

② skeckth BMD & SFD (give EI is constant)



$M_A = M_C = 0$

$$M_A \cdot L_1 + 2M_B(L_1+L_2) + M_C L_2 + \frac{6A_1\bar{x}_1}{L_1} + \frac{6A_2\bar{x}_2}{L_2} = 0$$

$$0 + 2M_B(l+l) + 0 + \left(\frac{\frac{6 \times wl^2}{8} \times l/2}{l}\right) + \left(\frac{\frac{6wl^2}{8} \times l/2}{l}\right) \qquad A_1 = 2/3 \times \frac{wl^2}{4} \times l$$

$$= 2M_B\, 2l + \frac{\frac{6 \times wl^2}{8} \times l/2}{l} + \frac{\frac{6 \times wl^2}{8} \times l/2}{l} = 0 \qquad = \frac{wl^2}{12}$$

$$\bar{x}_1 = \bar{x} = \frac{l}{2}$$

$$= 4M_B l + \frac{6}{8} wl^2 = 0$$

$$4M_B l = -3/4\, wl^2$$

$$M_B = \frac{-3}{16} wl \quad \text{(Hogging)}$$

BMD

$\frac{wl}{4}$

$$\frac{\frac{3}{16}wl}{x} = \frac{l}{l/2} \qquad 3/16\, wl = 2x$$

$$x = \frac{3}{32} wl$$

$$\frac{3/16\, wl}{x} = 2 \qquad \frac{wl}{4} - \frac{3}{32} wl = \frac{5wl}{32} \quad \text{sagging}$$

SFD

Reactions

$R_A + R_B + R_C = 2W$

$\Sigma M_B = 0$

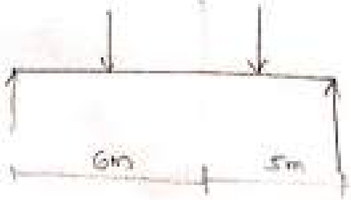
$R_A l - W l/4 + \frac{3}{16} Wl = 0$

$R_A \times l - \frac{WL \times 8}{2 \times 8} + \frac{3}{16} wl = 0$

$R_A L = 5Wl/16 = \frac{5W}{16}$

$\frac{5W}{16} + R_B + R_C = 2W$

- A continuous beam ABC is simply supported at, and loaded as shown in fig. find the support moment at BC and the reaction at the support. Sk SFD and BMD?

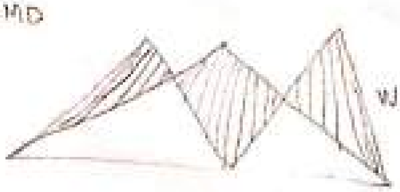


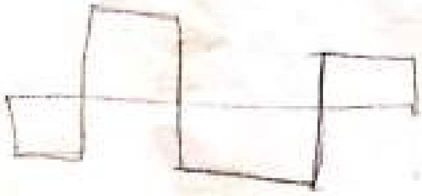
$A = 1/2 \; wl/4 \times l$

$= 1/2 \times \frac{8 \times 6}{4} \times 6$

$= \underline{\underline{36}}$

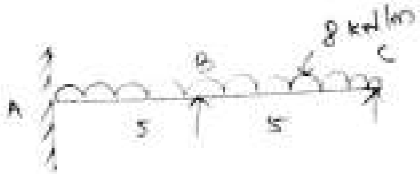
BMD





Note

If the continuous beam has more than 2 spans, then the theorem of 3 moments is to be applied first for span AB and BC. Then for BC and CD this will result in a set of simultaneous eqn and by solving these equations the unknown 'N' moments are calculated.



$M_0 = M_C = 0$

Apply clapeyron's theorem to span A

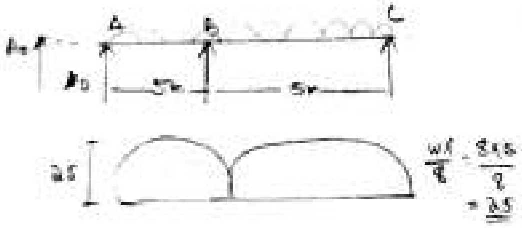
$M_0 L_0 + 2M_A(0+5) + M_B \cdot 5 + \frac{6 \times 0 \times 0}{0} + \frac{6 A_1 \bar{x}_1}{5}$

$0 + 10M_A + 5M_B + 249.99 = 0$

$0 + 10M_A + 5M_B + 250 = 0$

$2M_A + M_B + 50 = 0$

$2M_A + M_B = -50$ — (1)



$A_0 = 0$

$A_1 = \frac{2}{3} \frac{wl^2}{8} \cdot l$

$= \frac{2}{3} \times 25 \cdot 5$

$= 83.33$

$\bar{x}_1 = 2.5$

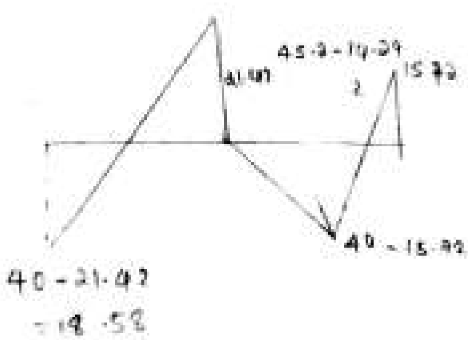
Applying clapeyron's theorem to AB and BC

$M_A \cdot 5 + 2M_B(5+5) + M_C \cdot 5 + \frac{6 \times 83.33 \times 2.5}{5}$

$5M_A + 20M_B + 500 = 0$

$5(M_A + 4M_B + 100) = 0$

$M_A + 4M_B = -100$ — (2)



(2) $\times 2 \Rightarrow 2M_A + 8M_B = -200$ — (3)

(3) & (1) subtract

$2M_A + 8M_B = -200$

$2M_A + M_B = -50$

$- 7M_B = -150$

$\therefore M_B = -21.42$ kNm Hogging

$M_A = -14.29$ kNm Hogging

~~MA + 4MB = -100~~

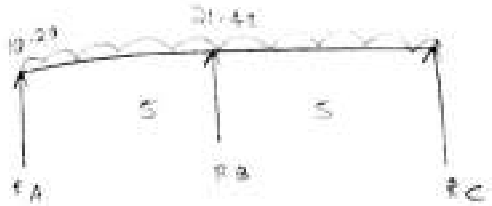
~~MA = -100 -~~

~~MA + 4x - 150 = 140~~

~~MA + 600 = 140~~

SFD

$R_A + R_B + R_C = 0$



$\Sigma M_B = 0$

$R_C \cdot 5 + 21.42 - 8 \times 5 \times 2.5 = 0$

$R_C \cdot 5 + -78.58 = 0$

$R_C \cdot 5 = +78.58$

$R_C = \dfrac{78.58}{5}$

$= \underline{15.72}$

$\Sigma M_A = 0$

$R_C \cdot 10 + R_B \times 5 - 8 \times 10 \times 5 + 14.29 = 0$

$15.72 \times 10 + 5 R_B - 400 + 14.29 = 0$

$157.2 + 5 R_B - 385.71 = 0$

$157.2 + 5 R_B = 385.71$

$5 R_B = 385.71 - 157.2$

$5 R_B = 228.51$

$R_B = \dfrac{228.51}{5}$

$= \underline{45.7 \text{ kN}}$

$R_A = 80 - (15.72 + 45.7)$

$= \underline{18.58 \text{ kN}}$

Moment Distribution Method (Hardy Cross Method)

A method to analys statically in determinate beams and frameds. It is developed by pro: Hardy cross also known as hardly cross method.

* Stiffnes factor (K)

The moment required to produce unit rotation

$$K = \frac{M}{\theta}$$

$M \rightarrow$ moment

$\theta \rightarrow$ Rotation

Note:-

For fixed end, $K = \frac{4EI}{L}$

For hinged end, $K = \frac{3EI}{L}$

Carry over Moment [CO]

The moment induced at the far end due to the distributed moment at the near end, the far end being fixed.

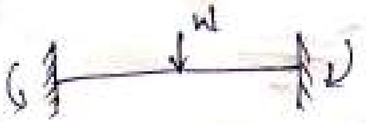
Distribution factor (DF)

The ratio of the stiffnes factor of the member being considerd to the sum of stiffness of all members meeting at that perticular continuous joint

$$DF = \frac{K}{\Sigma K}$$

Fixed End Moments (FEM)

Sign convertion

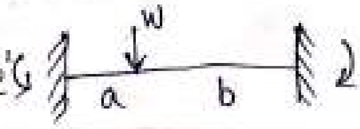


$M = +\frac{Wl}{8}$

$M = -\frac{wl^2}{12}$

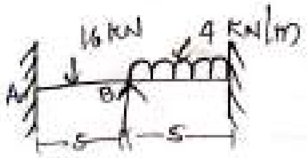
$M = \frac{wl^2}{12}$

$M = -\frac{W a \cdot b^2}{l^2}$



$M = \frac{W b \cdot a^2}{l^2}$

(6)



1. Assume all supports as fixed, 2. calculate FEM

$M_{FAB} = -\frac{Wl}{8} = \frac{-16\times5}{8} = -10 \text{ kN m}$

$M_{FBA} = \frac{Wl}{8} = \frac{16\times5}{8} = 10 \text{ kNm}$

$M_{FBC} = \frac{-wl^2}{12} = \frac{-4\times5^2}{12} = -8.33 \text{ kN m}$

$M_{FCB} = 8.33 \text{ kNm}$

3. Calculate BM at midspan of each span AB.

AB $= M = \frac{Wl}{4} = \frac{16\times5}{4} = 20 \text{ kNm}$

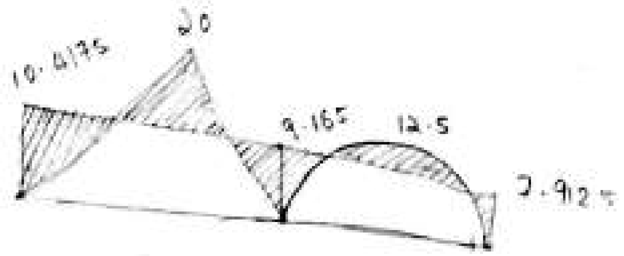
Span BC $= M = \frac{wl^2}{8} = \frac{4\times5^2}{8} = 12.5 \text{ kNm}$

4 Calculate Distribution factor

JOINT	member	K		DF
B	* BA	$\frac{4EI}{L} = \frac{4EI}{5}$	$-\frac{4EI}{5} + \frac{4EI}{5}$	* $\frac{4EI/5}{8EI/5}$ $= \frac{1}{2}$
	* BC	$\frac{4EI}{L} = \frac{4EI}{5}$	$= \frac{8EI}{5}$	* $\frac{4EI/5}{8EI/5}$ $= \frac{1}{2}$

5 calculate moment distributions

A		B	B		C	Joint
0		0.5	0.5		0	DF
-10		10	-8.33		8.33	FEM
		-0.835	-0.835			
-0.4175					-0.4175	CO
-10.4175		9.165	-9.165		7.9125	

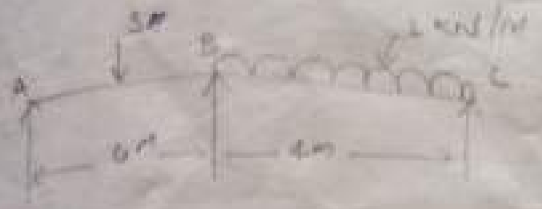


~~8.33~~
$-8.33 + 10$
$= 1.67$
$= 1.67 \times \frac{1}{2}$
$= -0.835$

Distribute

$\frac{-0.835}{2}$

8



1 Assume all support fixed

2 calculate FEM

$$MFAB = \frac{-wl}{8} = \frac{5\times 6}{8} = -3.75\ kNm$$

$$MFBA = \frac{+wl}{8} = \frac{5\times 6}{8} = 3.75\ kNm$$

$$MFBC = \frac{-wl^2}{12} = \frac{-5\times 4^2}{12} = -2.66$$

$$MFCB = 2.66\ kNm = 2.66$$

3 calculate BM at midspan A

$$\text{span } AB = \frac{wl}{4} = \frac{5\times 6}{4} = 4.5$$

$$\text{span } BC = \frac{wl^2}{8} = 4$$

$$= \frac{6EI + 6EI}{5\ 8} \quad \frac{10EI}{8} \quad 5/4$$

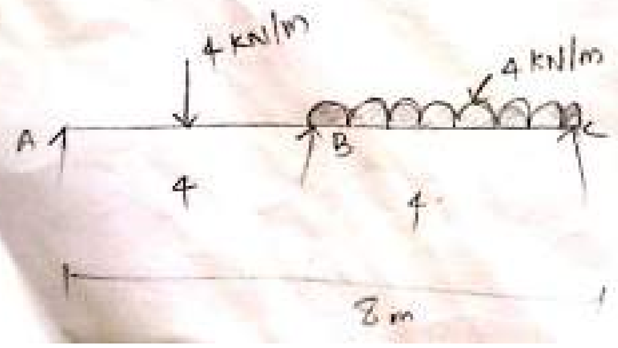
4 DF?

Joint	member	k	ΣK	DF
B	BA	$\frac{3EI}{L} = \frac{3EI}{6} = \frac{EI}{2}$	$= \frac{EI}{2} + \frac{3EI}{4} = \frac{5}{4}EI$	$\frac{EI/2}{5/4\,EI} = 2/5 = 0.4$
	BC	$\frac{3EI}{L} = \frac{3EI}{4}$		$\frac{3EI/4}{5/4\,EI} = 0.6$

5 calculate moment distribution

A	B		C	Joint
0	0.4	0.6	0	DF
-3.75	3.75	-2.67	2.67	FEM
3.75			-2.67	Release A, C
0	1.875	-1.335	0	
0	5.625	-4.005	0	Corrected FEM
	-0.648	0.972		
0	4.977	-4.977	0	

- A contineous beam ABC, 8m long rest on 3 s.s A, b, and c such that AB = BC = 4 m. It carries a point load of 4 kN at the mid length of AB and udl of 4 kN/m for BC. Draw the BMD using moment distribution method.? Draw SFD?



Scanned by CamScanner

1 assume all support fixed

2 calculate FEM

$$MFAB = -\frac{wl}{8} = \frac{4\times4}{8} = -2$$

$$MFBA = \frac{wl}{8} = \frac{4\times4}{8} = 2$$

$$MFBC = \frac{+wl^2}{12} = \frac{4\times4^2}{12} = -5.33$$

$$MFCB = \frac{wl^2}{12} = \frac{4\times4^2}{12} = +5.33$$

3 calculate BM at span ~~A~~ (mid)

$$MAB = \frac{wl}{4} = \frac{4\times4}{4} = 4$$

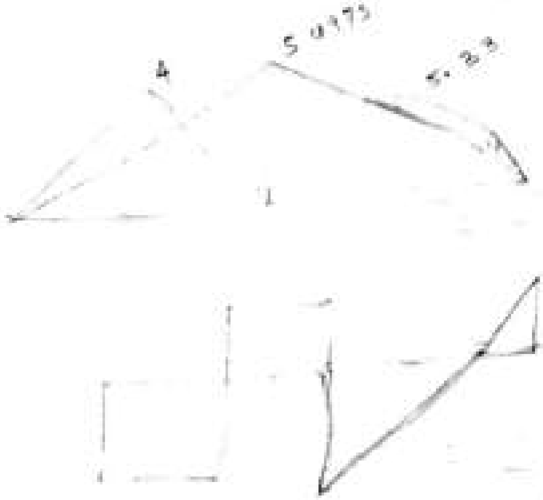
$$MBC = \frac{wl^2}{8} = \frac{4\times4^2}{4} = 8$$

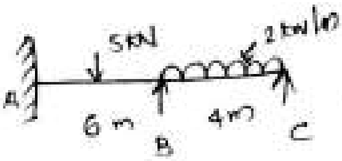
4 DF

Joint	Member	K	ΣK	DF
B	BA	$\frac{3EI}{L} = \frac{3EI}{4}$	$= \frac{3EI}{4} + \frac{3EI}{4}$	$\frac{\frac{3EI}{4}}{\frac{6EI}{4}} = \frac{1}{2}EI = 0.5$
	BC	$\frac{3EI}{L} = \frac{3EI}{4}$	$= \frac{6EI}{8} = \cancel{3/4EI}$	

5 calculat moment distribution

A	B		C	Joint
0	0.5	0.5	0	
-2	2	-5.33	5.33	initial FEM
2			-5.33	
0	1	-2.665	0	Release A&C
	3	-7.995	0	corrected FEM
	2.4975	2.4975		
	5.4975	-5.4975		



(2) 

1 Assume all supports are fixed

2 calculate FEM

$$M_{FAB} = \frac{-wl}{8} = \frac{5\times 6}{8} = -3.75\,km$$

$$M_{FBA} = \frac{wl}{8} = \frac{5\times 6}{8} = 3.75\,KM$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{2\times(4)^2}{12} = 2.67$$

$$M_{FCB} = \frac{wl^2}{12} = \frac{2\times(4^2)}{12} = 2.67$$

3 calculate Bm at span (mid)

$$M_{AB} = \frac{wl}{4} = \frac{5\times 6}{4} = 7.5$$

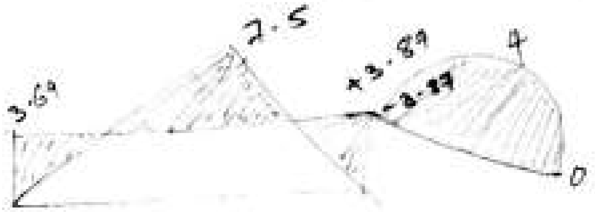
$$M_{BC} = \frac{wl^2}{8} = \frac{2\times(4)^2}{8} = 4$$

4 DF

joint	member	K	ΣK	DF
B	AB	$\frac{4EI}{L} = \frac{4EI}{6}$	$\frac{4EI}{6} + \frac{3EI}{4} = \frac{17EI}{12}$	$\frac{4EI/6}{17EI/12} = \frac{8}{17}EI = 0.470$
	BC	$\frac{3EI}{L} = \frac{3EI}{4}$		$\frac{3EI/4}{17EI/12} = 0.529$

5 calculate moment distribution.

A	B		C	Joint
0	0.470	0.53	0	DF
-3.75	3.75	-2.67	2.67	Initial FEM
			-2.67	Release C
		-1.335	0	
-3.75	3.75	-4.005	0	corrected FEM
		$(-\cancel{7.66})$ 0.255 0.255		
	0.1199	0.135		
0.0599				
-3.6901	3.87	-3.87	0	



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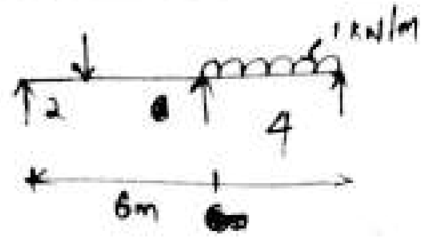


1 Assume all supports are fixed

2 calculate FEM

$$M_{AB} = \frac{-wab^2}{l^2} = \frac{-3 \times 2 \times 4^2}{6^2} = 45\ kN\,m$$

$$M_{BA} = \frac{-3 \times 4 \times 2^2}{6^2} = 1.53\ kN$$

3

$$M_{FBC} = \frac{-wl^2}{12} =$$

A solid $\square^{lar}$ coloumn of length 4 m and having a cross section of 200 x 100 mm if the ends of the members are hinged ~~are~~, find elux's in cripping load take $E = 200 \times$ KN/m²

$$PG = \frac{\pi^2 EI}{le^2}$$

$E = 200 \times 10^3$ N/mm²

CS = 200 x 100 mm

$I =$

$l = 4m$
$= 4000mm$

$$I = \frac{bd^3}{12}$$

$$= \frac{\pi^2 \times 2 \times 10^3 \times \frac{200 \times 100^3}{12}}{4000^2}$$

$$= 20561.267 \text{ N/mm}^2$$

A ~~estacma~~ hollow alloy tube 4 m long anes external & internal dia 40, 25 restly find Euler's buckling load. when bothe ends are pinned $E = 70 \times 10^3$ N/mm²

$l = 4$ m
$= 4000$ mm

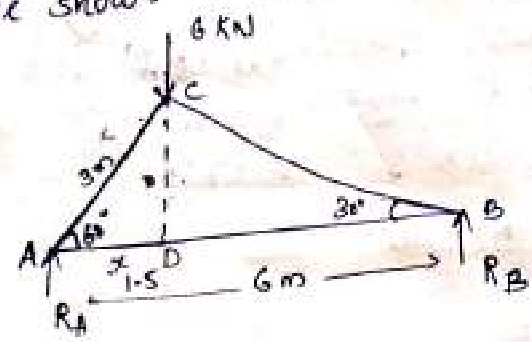
$$\frac{\pi D^4}{64} - \frac{\pi d^4}{64}$$

$$PE = \frac{\pi^2 EI}{le^2}$$

$$= \frac{\pi^2 \times 70 \times 10^3 \times \left(\frac{\pi \times 40^4}{64} - \frac{\pi\, 25^4}{64}\right)}{4000^2}$$

$= 40598.14$

figure shows a truss



$\cos 60 = \frac{AD}{3}$

$AD = \cos 6...$

$= 1.5$

Reactions

$AB = 6D$

~~tan AD = 6D / AB~~

$R_A + R_B = W_1$

~~tan A = AC / AD~~

$\Sigma M = 0$

~~AD = AC / tan A~~

$R_A + R_B = 6$

$= \frac{3}{\tan 60}$

$\Sigma f_x = 0$

$H_A = 0$

$\Sigma M_A = \cancel{6 \times 1.5 + 4.5}\ \cancel{6 \times 4.5}$

~~Sin 60 = AD / AB~~

$= R_A \times 6 = 6 \times 4.5$

~~AD = AC / Sin 60~~

$R_A = \frac{27}{6}$

$= \frac{3}{\sin 60}$

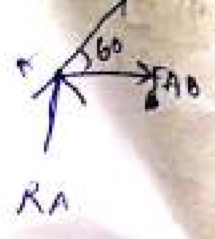
$= \underline{\underline{4.5}}$ kN

$R_B = 6 - 4.5$

$= \underline{1.5}$ kN

~~$\Sigma f_y = 0$~~ ~~4.5~~

Joint A

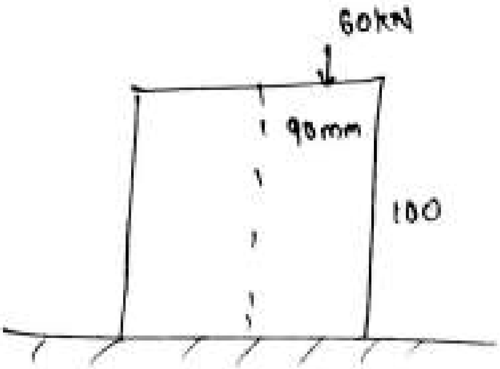


$F_y = 0$ 4.

A short column 100 x 100 mm is subjected to ... eccentricity load of 60 KN at eccentricity ... 40 mm in plane bisecting to opposite phase

find the maximum excentri

$e = 40$



$F_{max} = P/A + \frac{M}{Z}$

$F_{min} = P/A - \frac{M}{Z}$

$M = P \cdot e$

$= (60 \times 10^3)$

$= 240000$

$Z = \frac{I}{y}$

$= \frac{bd^2}{6}$

$= \frac{100 \times 100^2}{6}$

$=$

- A hollow circular colomn having external & int diameters of 350 mm & 300 mm respectively carries a vertical load of 80 kN

- A rectangular masonry dam having water concret vertical phace is 5m ht and 2.5 m wide.

find

(1) The resultand rest ~~to~~ (2) The point were the resultard cutts the base?

(3) Intensitys of maxm. & minim pressure across the section?

Specific weight of masonry = 20 kN/m³

water level coinsides with the top of the dam.