1. Matrices and Determinants

I. *Equality, Addition and Scalar Multiplication of Matrices*

1. Find the values of a, b, c and d, given that $\begin{bmatrix} 2a & a+3b \\ 5-c & d \end{bmatrix} = \begin{bmatrix} 4 & 11 \\ 7 & 0 \end{bmatrix}$.

2. Find the values of a, b, c and d, if $\begin{bmatrix} a+3 & 3a-2b \\ 3a-c & a+b+c \end{bmatrix} = \begin{bmatrix} 2 & -7+2b \\ b+4 & 2a \end{bmatrix}$.

3. If $A = \begin{bmatrix} 4 & 3 \\ 1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$ find A –3B.

4. If $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -3 \\ 0 & 1 \end{bmatrix}$ find 2A + 3B.

5. If $A = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 \\ 0 & 2 \end{bmatrix}$ find A – 2B.

6. If $A = \begin{bmatrix} 2 & 5 \\ 6 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -4 \\ 0 & -3 \end{bmatrix}$ find 2A + B.

7. If $A = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix}$ find 2A + B.

8. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$, find 2A – 3B.

9. Find A, if $A = 2\begin{bmatrix} 2 & 3 \\ -5 & 1 \end{bmatrix} - 3\begin{bmatrix} 7 & 10 \\ -3 & -11 \end{bmatrix}$.

10. If $A - \begin{bmatrix} 3 & 5 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, find A.

11. Find A, if $A + \begin{bmatrix} 5 & 6 \\ 7 & 0 \end{bmatrix} = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

12. If $A - B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ and $A + B = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$, find A and B.

13. If $A + 2B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$ and $2A + 3B = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix}$, find A and B.

14. Solve: $2A + 3B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $3A - 2B = \begin{bmatrix} 5 & 4 & 3 \\ 2 & 1 & 0 \end{bmatrix}$.

II. *Matrix Multiplication*

1. If $A=\begin{bmatrix}1 & -1\\ 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}0 & 1\\ -1 & 2\end{bmatrix}$, find AB.

2. If $A=\begin{bmatrix}2 & 3\\ 0 & -1\end{bmatrix}$, find A^2.

3. If $A=\begin{bmatrix}1 & 4 & 3\\ -4 & 0 & 1\end{bmatrix}$ $B=\begin{bmatrix}1 & 2\\ 0 & 5\\ -1 & 1\end{bmatrix}$, show that $AB \neq BA$.

4. If $A=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 1 & 0 & 0\end{bmatrix}$ and $B=\begin{bmatrix}2 & -1 & 3\\ 0 & 1 & 2\\ 0 & 2 & 3\end{bmatrix}$, Find AB and BA.

5. If $A=\begin{bmatrix}3 & 1 & 2\\ -1 & 2 & 3\\ 2 & -5 & 7\end{bmatrix}$ and $B=\begin{bmatrix}-2 & 4 & 1\\ 3 & -1 & 2\\ 4 & 1 & 3\end{bmatrix}$, find AB and BA.

6. If $A=\begin{bmatrix}2 & 3 & 4\\ 1 & 2 & 3\\ -1 & 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3 & 0\\ -1 & 2 & 1\\ 0 & 0 & 2\end{bmatrix}$, find AB and BA and show that $AB \neq BA$.

7. If $A=\begin{bmatrix}0 & 1 & -1\\ 2 & 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}1 & 2 & 3\\ -1 & 0 & -2\end{bmatrix}$ and $C=\begin{bmatrix}1 & 2 & 0 & -1\\ 0 & 1 & 2 & 3\\ 1 & 0 & 1 & 2\end{bmatrix}$ show that (A + B) C = AC + BC.

8. If $A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & 2 & 3\\ 3 & 0 & 1\end{bmatrix}$ and $C=\begin{bmatrix}2 & -1 & -1\\ -2 & 2 & 3\end{bmatrix}$ verify that A (B + C) = AB + AC.

9. If $A=\begin{bmatrix}4 & 1 & 0\\ 2 & 1 & 5\\ -1 & 1 & 0\end{bmatrix}$, $B=\begin{bmatrix}2 & -1 & 3\\ 1 & 2 & 1\\ 2 & -2 & 5\end{bmatrix}$ and $C=\begin{bmatrix}2 & 1 & 3\\ -1 & 1 & -1\\ 2 & 1 & 0\end{bmatrix}$, verify that A (B + C) = AB + AC.

10. If $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$, find $A^2 - 3A + 5I$.

11. If $A=\begin{bmatrix}1 & 2 & 6\\ 7 & 4 & 10\\ 1 & 3 & 5\end{bmatrix}$, find $A^2 - 3A + I$.

12. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, show that $A^2 - 4A - 5I = 0$.

13. If $A(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, show that $A(\theta)\cdot A(\theta') = A(\theta+\theta')$.

14. If $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$, prove that $A^2 = B^2$.

III. *Transpose of a matrix*

1. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}0 & -2\\ -3 & -3\end{bmatrix}$ find $(A + B)^T$.

3. If $A=\begin{bmatrix}2\\ -1\end{bmatrix}$ and $B=\begin{bmatrix}0 & -1\\ 3 & 1\end{bmatrix}$, find A^TB.

4. If $A=\begin{bmatrix}3 & 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}0 & 2 & -1\end{bmatrix}$, find B^TA.

5. If $A=\begin{bmatrix}2 & 3 & 0\\ 3 & -1 & 1\end{bmatrix}$, find AA^T.

6. If $A=\begin{bmatrix}2 & 3\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}3 & 4\\ 2 & 1\end{bmatrix}$, verify that $(A + B)^T = A^T + B^T$.

7. If $A=\begin{bmatrix}2 & 3\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}3 & 4\\ 2 & 1\end{bmatrix}$, prove that $(AB)^T = B^TA^T$.

8. If $A=\begin{bmatrix}1 & 2 & 3\\ 0 & -2 & 6\\ 1 & 3 & 2\end{bmatrix}$, compute $A + A^T$ and show that $A + A^T$ symmetric.

9. If $A=\begin{bmatrix}1 & 0 & 5\\ -2 & 1 & 6\\ 3 & 2 & 7\end{bmatrix}$, show that $A\,A^T$ and show that $A^T\,A$ are symmetric.

10. If A is a square matrix, show that $A + A^T$ is symmetric and $A - A^T$ is skew symmetric.

11. Show that every square matrix can be expressed as the sum of symmetric and skew symmetric matrices.

12. Express the matrix $\begin{bmatrix}1 & 4 & 5\\ 2 & 2 & 3\\ 3 & 1 & 0\end{bmatrix}$ as the sum of symmetric and skew symmetric matrices.

13. Express the matrix $\begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & 6 \\ 3 & 2 & 7 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices.

14. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 & 0 \\ 2 & -3 & 1 \\ 1 & 1 & -1 \end{bmatrix}$, show that $(AB)^T = B^T A^T$.

IV. ***Determinants***

1. Find the value of $\begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix}$

2. Solve for x, if $\begin{vmatrix} x & 12 \\ 3 & x \end{vmatrix} = 0$

3. Solve for x, if $\begin{vmatrix} x & 3 \\ 5 & 2x \end{vmatrix} = \begin{vmatrix} 5 & -4 \\ 5 & 3 \end{vmatrix}$

4. Solve for x, if $\begin{vmatrix} 2 & 3 & 5 \\ 2 & x & 5 \\ 3 & -1 & 2 \end{vmatrix} = 0$.

5. Solve for x, if $\begin{vmatrix} 3x & 7 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix}$

6. Solve for x, if $\begin{vmatrix} x & -1 \\ -1 & x \end{vmatrix} = \begin{vmatrix} 2 & -x \\ 2x & -1 \end{vmatrix}$

7. Solve for x, if $\begin{vmatrix} 3 & 1 & 9 \\ 2x & 2 & 6 \\ x^2 & 3 & 3 \end{vmatrix} = 0$.

8. Solve for x, if $\begin{vmatrix} 1 & 2 & 3 \\ 2 & x & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$.

9. Solve for x, if $\begin{vmatrix} 3x & -2 & 0 \\ 3 & 1 & 0 \\ 0 & 10 & 1 \end{vmatrix} = 0$.

9. If $\begin{vmatrix} 2 & 1 & x \\ 3 & -1 & 2 \\ 1 & 1 & 6 \end{vmatrix} = \begin{vmatrix} 4 & x \\ 3 & 2 \end{vmatrix}$, find x.

V. ***Application of Determinants***

Solve using Cramer's rule :

1. $x+2y-z=-3,\ 3x+y+z=4,\ x-y+2z=6$
2. $2x-y+z=3,\ x+4y+3z=-3,\ 3x-2y-z=5$
3. $x+y-z=4,\ 3x-y+z=4,\ 2x-7y-3z=-6$
4. $2x+3y-z=5,\ x-2y+3z=6,\ 3x-y+2z=7$
5. $x+y+z=3,\ 2x+3y+z=-6,\ x-y-z=-3$
6. $x-y+2z=2,\ 2x+y-z=2,\ x+y+z=3$
7. $x+y+z=6,\ x+2y-z=2,\ 2x+y+z=7$
8. $x+2y+z=1,\ -x+y+z=-1, x+y-z=1$
9. $3x-y+2z=8,\ x+y+z=2,\ 2x+y-z=-1$
10. $x+2y+3z=6,\ 2x+4y+z=7, 3x+2y+9z=14$
11. $2x+3y+z=11,\ x+y+z=6,\ 5x-y+10z=34$
12. $x+2y+3z=11,\ 2x-y+4z=13,\ 3x+4y-5z=3$
13. $2a-3b+c=-1,\ a+4b-2c=3,\ 4a-b+3c=11$

Find the value of *k*/*p*, if the system of equations is consistent:

1. $x+y+1=0,\ x+2y+1=0,\ 2x+3y+k=0$ 2. $kx+3y-5=0,\ 5x-y+3=0,\ 7x+ky-2=0$

3. $2x+3y+9=0,\ 4x+ky+13=0,\ kx-2y-25=0$

Eliminate x and y from the following system of equations:

1. $ax+by+c=0,\ bx+cy+a=0,\ cx+ay+b=0$ 2. $ax+by=0,\ ay+b=0,\ bx+a=0$

VI. ***Inverse of a matrix:***

1. Find the inverse of $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

2. Find A^{-1}, if $A=\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

3. Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix}$

4. Find the inverse of the matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}$

5. Find the inverse of the matrix $A=\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

6. Find the inverse of the matrix $A=\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

7. Find the inverse of the matrix $A=\begin{bmatrix} 2 & -1 & -3 \\ 0 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix}$

8. Find the inverse of the matrix $A=\begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 2 \\ 0 & -2 & 1 \end{bmatrix}$

9. Find the inverse of the matrix $A=\begin{bmatrix}3 & -2 & 3\\ 2 & 1 & -1\\ 4 & -3 & 2\end{bmatrix}$

10. Find the inverse of the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 2\\ 2 & 4 & 1\end{bmatrix}$

11. If $A=\begin{bmatrix}5 & 3\\ 2 & 2\end{bmatrix}$ and $\begin{bmatrix}7 & 5\\ 4 & 3\end{bmatrix}$, show that $(AB)^{-1}=B^{-1}A^{-1}$

12. If $P=\begin{bmatrix}1 & 2\\ 4 & 9\end{bmatrix}$ and $B=\begin{bmatrix}3 & 4\\ 2 & 1\end{bmatrix}$, show that $(PQ)^{-1}=Q^{-1}P^{-1}$

13. If $A=\begin{bmatrix}4 & 1\\ 6 & 5\end{bmatrix}$ find A^{-1} and show that $AA^{-1}=A^{-1}A=I$

VII. ***Application of inverse of a matrix***

Solve the following system of equations by finding the inverse of the coefficient matrix:

1. $3x+y-z=3,\ x-y-z=-1,\ x+y+z=3$
2. $3x+y-z=3,\ -x+y+z=1,\ x+y+z=3$
3. $x+y+z=0,\ -x+y+z=2, x+y-z=-2$
4. $3x-2y+3z=4,\ 2x+y-z=2, 4x-3y+2z=3$
5. $x-y+z=4,\ 2x+y-3z=0, x+y+z=2$
6. $2x+y+z=1,\ x-2y-z=\frac{3}{2}, 3y-5z=9$