

1. In any $\triangle ABC$, prove that $(b+c)\sin\frac{A}{2} = a\cos\left(\frac{B-C}{2}\right)$.
2. In any $\triangle ABC$, prove that $(a+b)\sin\frac{C}{2} = c\cos\left(\frac{A-B}{2}\right)$.
3. In any $\triangle ABC$, prove that $(a-b)\cos\frac{C}{2} = c\sin\left(\frac{A-B}{2}\right)$.
4. In $\triangle ABC$, prove that $\sum a(\sin B - \sin C) = 0$.
5. In any $\triangle ABC$, $A = 45^\circ$, $B = 60^\circ$, $a = 5\text{cm}$, find b .
6. In any $\triangle ABC$, $A = 30^\circ$, $C = 45^\circ$, $a = 2\text{cm}$, find c .
7. Prove that $2(bc\cos A + ca\cos B + ab\cos C) = a^2 + b^2 + c^2$.
8. Prove that $bc\cos A + ca\cos B + ab\cos C = \frac{a^2 + b^2 + c^2}{2}$.
9. Show that $a(b\cos C - c\cos B) = b^2 - c^2$.
10. In $\triangle ABC$, Prove that $R(a^2 + b^2 + c^2) = abc(\cot A + \cot B + \cot C)$
11. In any $\triangle ABC$, prove that $a^2 + bc = b^2 + c^2$, if $A = 60^\circ$.
12. Find the third side of a triangle, given $b = 2\text{cm}$, $c = 3\text{cm}$ and $A = 60^\circ$.
13. In any $\triangle ABC$, $b = \sqrt{3}$, $c = 1$, $A = 30^\circ$, find a .
14. In any $\triangle ABC$, $a = \sqrt{3} + 1$, $c = 2$, $B = 30^\circ$, find b .
15. Prove that $(b+c)\cos A + (c+a)\cos B + (a+b)\cos C = a+b+c$.
16. Show that $a(b^2 + c^2)\cos A + b(c^2 + a^2)\cos B + c(a^2 + b^2)\cos C = 3abc$.